GHOST MODES IN IMPERFECT WAVEGUIDES

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Ghost Modes in Imperfect Waveguides*

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Summary—**Attention is called to the existence of microwave resonances which have the unusual property that they are nonradiating, and thus have high Q, in spite of the fact that the fields are not enclosed completely by metallic walls. These "ghost modes" represent fields which are derived from those of the usual waveguide modes, but are "trapped" in the vicinity of imperfections in the waveguide. Their resonant frequency is slightly lower than the corresponding cutoff frequency. If one attempts to use a waveguide in its lowest mode, over a range of frequencies which includes the ghosts of higher modes, complicated resonance effects may be observed which can cause trouble in some types of microwave circuits. The same phenomenon exists in any structure which has pass bands and rejection bands, such as periodic structures or crystals.**

Introduction

The phenomenon described herein explains certain curious and troublesome effects observed in waveguides operating close to the cutoff frequency of a propagation mode, and it may have applications for waveguide filters. In addition, it has a certain educational value because of a mathematical analogy with the phenomenon of localized bound states due to imperfections in crystals.

Ghost Modes Due to a Dielectric

To illustrate the simplest case, and one which can be analyzed completely, consider the structure of Fig. 1. A dielectric disk of thickness d, dielectric constant ϵ, is placed in a cylindrical waveguide of radius a. Let the center of the disk be the origin of a cylindrical coordinate system $(r\theta z)$. A possible electromagnetic field is one whose transverse structure is that of the TM_{01} mode, derived from the field component

$$E_z = \begin{cases} BJ_0(k_1 r) \exp\left[-(k_1^2 - k^2)^{1/2} z\right], & z > \dfrac{d}{2} \\ AJ_0(k_1 r) \cos\left[(\epsilon k^2 - k_1^2)^{1/2} z\right], & |z| < \dfrac{d}{2} \\ BJ_0(k_1 r) \exp\left[+(k_1^2 - k^2)^{1/2} z\right], & z < -\dfrac{d}{2} \end{cases} \cdot \quad (1)$$

In (1),

$$k_1 = \frac{2.405}{a} = \frac{\omega_c}{c}, \qquad k = \frac{\omega}{c},$$

where ω_c and ω are, respectively, the TM_{01} cutoff frequency and an operating frequency to be determined presently. Matching solutions across the dielectric-air interfaces, we obtain the condition

$$\tan\left[(\epsilon k^2 - k_1^2)^{1/2}\frac{d}{2}\right] = \epsilon\left[\frac{k_1^2 - k^2}{\epsilon k^2 - k_1^2}\right]^{1/2}. \quad (2)$$

For any $\epsilon > 1$, (2) has at least one solution for k with $k < k_1$. If $(\epsilon - 1)^{1/2} k_1 d < 2\pi$, only one such solution exists. We consider, in particular, the case where ϵ is appreciably greater than unity and $k_1 d \ll 1$. Then k as determined from (2) is very close to k_1, and it will be a good approximation to set

$$(\epsilon k^2 - k_1^2)^{1/2} d \simeq (\epsilon - 1)^{1/2} k_1 d \ll 1,$$

so that (2) reduces to the following formula for ω:

$$\omega^2 \simeq \omega_c^2\left[1 - \left(\frac{\epsilon - 1}{\epsilon}\frac{k_1 d}{2}\right)^2\right]. \quad (3)$$

The field derived from (1) thus represents a resonant mode with resonant frequency slightly below the cutoff frequency of the TM_{01} mode, in which the fields are localized to the vicinity of the dielectric disk. The electric field configuration is sketched in Fig. 1. The longitudinal extension of this ghost mode is described by the "$1/e$ distance"

$$x_0 = \frac{1}{(k_1^2 - k^2)^{1/2}} = \frac{\lambda_c}{2\pi(1 - \omega^2/\omega_c^2)^{1/2}} \quad (4)$$

where $\lambda_c = 2\pi/k_1 = 2.6a$ is the TM_{01} cutoff wavelength. Numerical values of (x_0/λ_c) are indicated in Fig. 2. It is seen that the ghost is surprisingly well localized; for example, in the case $\omega = 0.99\ \omega_c$, x_0 is less than 1.5 pipe diameters. If the waveguide extends a distance three or four times x_0 on either side of the dielectric disk, this ghost will be essentially nonradiating and will have a very high Q, limited only by losses in the waveguide walls and the dielectric.

Suppose this waveguide were being operated in the TE_{11} propagating mode, at frequencies near the TM_{01} cutoff frequency. This is a rather common condition in practice, since the TM_{01} cutoff frequency is only about 30 per cent higher than that of the TE_{11} mode. In a perfect waveguide the fact that we were close to the TM_{01} propagating range would not cause any trouble. However, the disk represents an "imperfection" in what would otherwise be a smooth waveguide. The slightest asymmetry in structure near the disk, or the slightest inhomogeneity in dielectric constant of the disk, can couple the ghost to the TE_{11} mode and cause a large absorption of energy at the frequency given by (3). The field of the ghost can attain a magnitude several hundred times that of the propagating TE_{11} mode, so that

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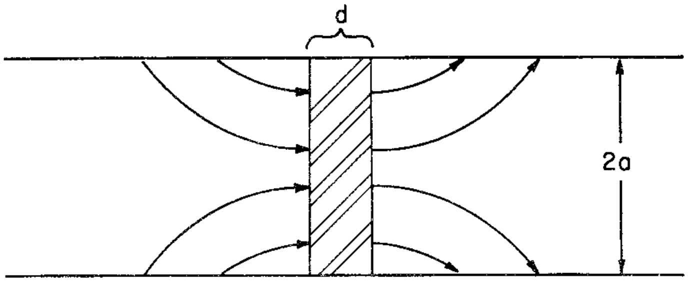

Fig. 1—TM_{01} ghost mode due to a dielectric disk.

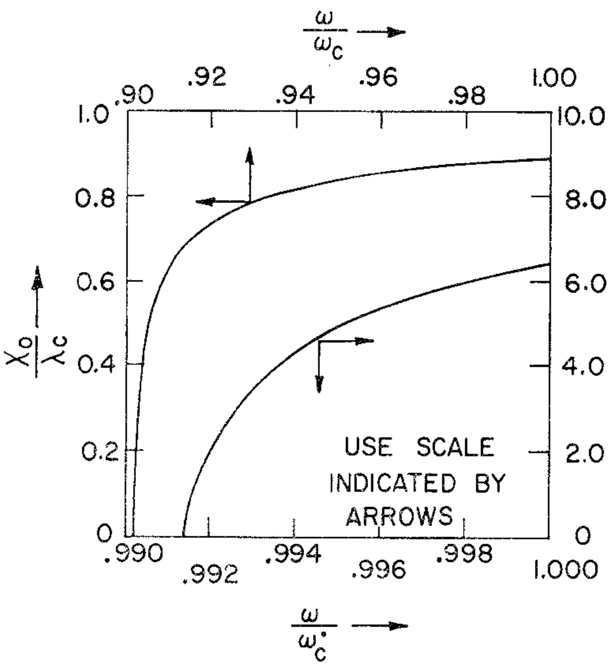

Fig. 2—"(1/e) distance" of ghost modes as a function of frequency.

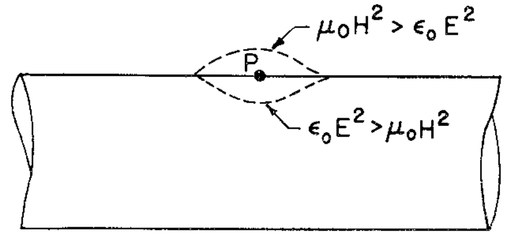

Fig. 3—Wall perturbations which create ghost modes.

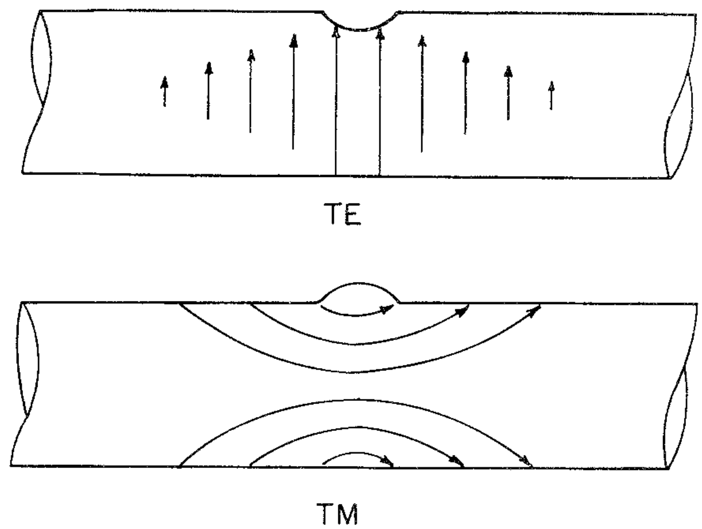

Fig. 4—Electric field lines of TE and TM ghost modes due to wall imperfections.

in a high-power device a ghost can cause breakdown. It was, in fact, the persistent failure of ceramic windows in the output waveguides of high-powered klystrons feeding the Stanford linear electron accelerator (carrying about 20 megw at 3 kmc), which led to the recognition of ghost modes. Spurious resonances in certain ferrite devices may also be traceable to this cause.

What was said for the TM_{01} mode evidently applies in general for the structure of Fig. 1; every propagation mode of the waveguide is accompanied by a ghost with the same transverse field pattern, localized to the vicinity of the disk, with a resonant frequency slightly below the corresponding cutoff frequency.

General Ghost Modes

The presence of a dielectric is not necessary for existence of ghosts: *any* localized imperfection in a waveguide will cause them to appear. Consider first a perfect, infinitely long waveguide with zero loss, excited in one of the propagation mode patterns at exactly its cutoff frequency. It is thus a large resonant cavity with resonant frequency ω_c. Let the electric and magnetic fields in the vicinity of any point P on the waveguide wall be E, H. Now if $\epsilon_0 E^2 > \mu_0 H^2$, push the wall in slightly at P, while if $\mu_0 H^2 > \epsilon_0 E^2$, pull out a small "blister" as in Fig. 3. According to the Slater perturbation formula [1], the resonant frequency of the cavity is always lowered by this change. But at any lower frequency than ω_c the fields will be attenuated exponentially as we move away from the imperfection; thus a ghost has been created, trapped to the vicinity of the imperfection, with a resonant frequency slightly lower than ω_c. Two examples are given in Fig. 4, in which only the electric field lines are sketched.

The Q of a ghost mode due to a small imperfection as in Fig. 4 is given to order of magnitude by

$$Q \simeq \frac{a}{\delta} = \frac{\text{waveguide radius}}{\text{skin depth}}, \tag{5}$$

from which it is seen that values in excess of 10^4 are easily attained. For the ghost of any mode other than the lowest one, there also will be a "loaded Q" involving the strength of coupling to propagating modes.

Crude estimates of the frequency and longitudinal extension of these ghosts, obtained from (4) and the Slater perturbation formula, are

$$\omega^2 \simeq \omega_c^2\left[1 - \left(\frac{\pi\alpha\delta V}{A\lambda_c}\right)^2\right], \tag{6}$$

$$x_0 \simeq \frac{A\lambda_c^2}{2\pi^2\alpha\delta V}. \tag{7}$$

Here A is the cross-sectional area of the waveguide, δV is the volume added or removed by the imperfection, and α a dimensionless quantity of the order of magnitude unity, given by

$$\alpha = \frac{|\mu_0 H^2 - \epsilon_0 E^2|}{<\mu_0 H^2 + \epsilon_0 E^2>_{Av}} \tag{8}$$

where the fields in the numerator are values near the imperfection, while the denominator is an average over a waveguide cross section passing through the imperfection [2].

From these formulas several general conclusions may be drawn. If the imperfection is so small that

$$\delta V < \frac{A\lambda_c}{100}, \tag{9}$$

then one may expect the ghost to be separated from the cutoff frequency by less than the bandwidth ω_c/Q, so that it does not appear as a well-resolved resonance, but only a slight broadening of the cutoff region. If, on the other hand,

$$\delta V > \frac{A\lambda_c}{100}, \tag{10}$$

the ghost will generally appear as a distinct resonance. The longitudinal extension x_0 of any well-resolved ghost will, from Fig. 2, never be more than a few times λ_c. The δV required by (10) is so large that the usual mechanical imperfections due to machining errors, inadvertent hammer blows, etc., do not cause separation of ghosts from the waveguide modes. However, installation of almost any kind of apparatus inside a waveguide will cause them to appear, as may the presence of any twist or bend.

Periodic Structures

What was said above for waveguides applies equally well to any periodic structure, such as those used in linear accelerators, traveling-wave tubes, or, in fact, any lumped-constant filter composed of a cascade of identical networks. By reasoning exactly like that in the preceding section, one can show that any localized imperfection in such a structure will cause a bound resonance to appear, with resonant frequency just outside the pass band of the structure. In this case, it may lie either above or below the pass band. Once again, the imperfection has to be quite large in order to cause separation of a well-resolved resonance, so that reasonable care in construction is sufficient to avoid troubles due to these ghosts.

There is an interesting, and pedagogically useful, analogy with certain well-known features of solid-state theory. Here a crystal represents a three-dimensional periodic structure, and solutions of the Schrödinger equation, in one-electron approximation, show an almost perfect mathematical analogy with those in the corresponding electrical problem [3]. In particular, the conduction bands of the crystal correspond to pass bands of the filter, and any localized imperfection in the crystal (such as a vacancy, interstitial atom, or impurity atom), results in at least one localized bound state, with an energy just above or just below the edge of a conduction band. In this way one can understand the creation of donor or acceptor impurity levels involved in n-type and p-type conductivity in semiconductors [4].

Further work on ghost modes, by M. Forrer and the writer, is being prepared for publication. Experimental results, universal curves for predicting the occurrence of ghosts, and discussion of their possible use in waveguide filters and mode convertors will be included.

Bibliography

[1] Slater, J. C. *Microwave Electronics*, New York: D. Van Nostrand Company, Inc., 1950, p. 81.

[2] The derivation of (6) and (7) is a bit tricky, since a small change in δV alters the "length" x_0 of the cavity. One must apply the Slater formula only to infinitesimal changes in δV, and integrate to obtain their cumulative effect. For this reason the frequency shift involves $(\delta V)^2$ rather than (δV), as one might at first expect.

[3] Brillouin, L. *Wave Propagation in Periodic Structures*, New York: McGraw-Hill Book Company, Inc., 1946.

[4] Kittel, C. *Introduction to Solid-State Physics*. New York: John Wiley & Sons, Inc., 1953, Ch. 14.