COMMENTS AND ADDENDA

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Comment on the Characteristic Time of Spontaneous Decay in Jaynes's Semiclassical Radiation Theory

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We examine the implication in the fact that, in a semiclassical theory of atomic structure proposed by Jaynes and Crisp, the initial state of an atom plays a critical rôle in determining its average lifetime in an excited state.

In a recent publication,[1] Jaynes and Crisp have studied the behavior of atoms within the framework of a semiclassical theory, which includes the effects upon the atom of the fields created by the atomic currents. They state that, in the absence of an applied field, an atom will spontaneously decay from an excited state with a characteristic time which is equal to the reciprocal of the Einstein A coefficient for the transition. The purpose of this note is to point out that statement is not entirely precise. To see this we note that the solution to the nonlinear density-matrix equation, for the diagonal matrix elements in a two-level decay, in a spontaneous transition with no applied fields can be written

$$\rho_{11}(t)=1/[\exp(-A_{21}(t-t_m))+1] \,, \tag{1}$$

$$\rho_{22}(t)=[\exp(A_{21}(t-t_M))+1]^{-1} \,, \tag{2}$$

where $\rho_{11}(t)+\rho_{22}(t)=1$

and $$t_m=A_{21}^{-1}[\ln(\rho_{22}(0)/\rho_{11}(0))] \tag{3}$$

is related to the initial state of the atom at $t=0$. If we temporarily neglect that part of the self-field which yields only a small frequency shift, the solution for the off-diagonal elements are given by

$$\rho_{12}(t)=\rho_{21}^{*}(t) = \left(\frac{\rho_{12}(0)}{\rho_{22}(0)}\right)\frac{\exp(-i\Omega_{12}t+A_{21}t/2)}{\exp[A_{21}(t-t_m)]+1} \,, \tag{4}$$

where $\rho_{nm}(0)\equiv C_n(0)C_m^*(0)$ and $C_n(0)$ are the initial values of the coefficients ($n=1,2,$) in the wavefunction of the atom which describes the transition process. We first note, that the value of t_m determines the point at which the maximum atomic-dipole moment occurs [the effective-dipole moment of the transition is proportional to $(\rho_{12}+\rho_{21})$]. In Jaynes's semiclassical theory, it is assumed that the expectation value of the dipole moment of the atom is responsible for the radiation process. Hence, an excited atom radiates slowly until its dipole moment grows to an appreciable value, and then begins to radiate its energy away very rapidly. While this characteristic behavior was duly noted by Jaynes and Crisp in their article, the role that the value of t_m (the point in time where the maximum dipole radiation occurs) plays in determining the average lifetime of the atom was not clarified. While it is certainly true that Eqs. (1), (2), and (4) imply that most of the transition energy is radiated during a time interval which is proportional to the reciprocal of the associated Einstein A coefficient A_{21}, this "characteristic energy transfer time" is *not* equal to the average lifetime of the atom. In particular, if we define the average lifetime of the atom as that time required for $\rho_{22}(0)\simeq 1$, at $t=0$, to decay to $1/e$th of its initial value, we find from Eq. (2) that

$$\tau_2 \sim (A_{21}^{-1})\ln[(1.718)\rho_{22}(0)/\rho_{11}(0)](\text{sec}). \tag{5}$$

Hence, we see that the initial values of $\rho_{22}(0)$

and $\rho_{11}(0)$ [where $\rho_{11}(0)+\rho_{22}(0)=1$] play a critical role in determining the atoms average lifetime, while the "characteristic energy transfer time" (occurring in some interval[2] of time Δt about t_m) is essentially independent of the atoms initial condition at $t=0$. The implication here is that there is a need to include an additional element into the theory, one which can account for the experimental fact that the average lifetime of an atom is essentially *independent* of its preparation. One possibility is that the presence of a "semiclassically described" vacuum state[3] might produce the required behavior when properly included in the associated nonlinear density-matrix equations of the theory.

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[1]E. T. Jaynes and M. D. Crisp, Phys. Rev. 179, 1253 (1969).

[2]Since the maximum-dipole moment occurs at $t=t_m$ in Eq. (4), the "time halfwidth" of the associated dipole radiation pulse about the maximum is given by

$$\Delta t \sim 2A_{21}^{-1}\ln\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right).$$

Hence the associated frequency halfwidth is $\Delta\omega \sim (\Delta t)^{-1} < A_{21}$, which is smaller than that predicted quantum electrodynamics.

[3]E. A. Uehling, Phys. Rev. 48, 55 (1935).

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Reply to Leiter's Comment

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Leiter[1] raises an important point which illustrates the need for more refined experiments before we could claim to understand the dynamics of spontaneous emission and other radiation processes. The same point was raised by Schawlow at the 1966 Rochester Coherence Conference, and answered in the ensuing discussion. We welcome the opportunity to clarify matters to a wider audience.

The semiclassical or "neoclassical" theory (NCT) in question was developed by the writer and his colleagues [2–7] with the following motivation. Our present quantum electrodynamics (QED) has not achieved any satisfactory final form; it contains many important "elements of truth," but is mixed up with clear "elements of nonsense." The divergence and other difficulties indicate, that at least one of its underlying principles must be modified; but for forty years we have lacked experimental clues suggesting where and how this should be done, and nobody has seen how to disentangle the truth from the nonsense.

A possible way out of this impasse is to try to construct alternative theories in which various objectionable features of QED are eliminated by *fiat*, and see whether they suggest new experiments capable of deciding among them. If some alternative theory could be shown to contain just one grain of truth that is not contained in present QED, then we would have the missing clue showing how QED must be modified.

NCT automatically removes all divergences arising from field quantization and infinite vacuum fluctuations, but retains the conventional Schrödinger equation to describe the behavior of matter. Although energy exchanges between field and matter then take place continuously, there is a strong tendency for this to occur in units of $\hbar\omega$, explained by NCT in a completely mechanistic and causal manner as a consequence of the equations of motion for matter – just as Planck and Schrödinger always believed must be true.

To the best of our knowledge, NCT agrees with existing experiments in every case where accurate calculations have been completed.[8] But the predictions always differ from those of QED in finer details on which we have as yet no experimental evidence. The case of spontaneous emission discussed by Leiter is one example of this. Considering for simplicity only two levels, when an atom is excited (for example, by electron impact) we have to expect that, in general, it will not be left in exactly the excited state ψ_2 at the moment of excitation $t=0$, but in some linear combination $\psi(0) = a_1\psi_1 + a_2\psi_2$, where ψ_1 is the ground state. In QED, we interpret $|a_2|^2$ as the probability that the atom is excited to the upper state, and each excited atom proceeds to radiate a spontaneous emission pulse with field amplitude at a given point of the form

$$Ce^{-At/2}\cos(\omega t+\theta)\,, \tag{1}$$

where $\hbar\omega = E_2 - E_1$, and A is the Einstein A coefficient for the transition. The total energy radiated in the pulse is $\hbar\omega$.

In NCT, the predicted spontaneous emission pulse is of the form

$$C' \operatorname{sech}\left[\tfrac{1}{2}A(t-t_m)\right]\cos[\omega(t-t_m)+\theta]\,, \tag{2}$$

where $C'^2 = \frac{1}{2}C^2$, and t_m is determined by the initial state through Leiter's Eq. (3) with $\rho_{22}(0) = |a_2|^2$, etc. The observed pulse, of course, consists only of the portion of this function for $t > 0$; and so NCT predicts a spontaneous emission pulse with a truncated hyperbolic secant envelope rather than an exponential one. Furthermore, the total energy radiated during the pulse is $\hbar\omega|a_2|^2 \leq \hbar\omega$. As Leiter notes, if $|a_2|^2$ is near unity, there is an appreciable delay time t_m before maximum emission is reached. For example, if $|a_2|^2 = [0.9;\ 0.99;\ 0.999]$, we find $At_m = [4.4;\ 9.2;\ 13.8]$, respectively. This behavior contradicts what we have all been taught in courses on quantum theory. The relevant question is: *Does it contradict experiment?*

The common methods of excitation – whether by collision or by absorption of radiation – are highly inefficient, i.e., the upper state attains an amplitude $|a_2| \ll 1$. But then t_m in Eq. (2) is negative, the cases $|a_2|^2 = [0.4;\ 0.1;\ 0.01]$ yielding $At_m = [-0.81;\ -4.4;\ -9.2]$, respectively. The emitted radiation, according to NCT, thus, consists only of the exponential tail of the hyperbolic secant pulse, in Eq. (2); since $\operatorname{sech} x \simeq 2e^{-x}$ for $x > 1$, this is of the same form as the QED pulse, in Eq. (1) except for a smaller amplitude.

Experiments on radiation from excited atoms have, for intensity reasons, necessarily observed only the net radiation from many atoms simultaneously. As long as the excitation mechanism is inefficient, $|a_2|^2 \ll 1$, these two theories would describe such experiments as follows. QED: A very small fraction of the atoms is excited by collision, and each one emits the full exponential pulse as in Eq. (1); NCT: Each atom, on collision, emits an exponential pulse of the shape given by Eq. (1), but with an amplitude proportional to the particular value of $|a_2|$ produced in the collision.

On either theory, the total radiation emitted and its spectral distribution are identical. QED predicts greater instantaneous intensity fluctuations; but statistical calculations by Dr. Charles Owen and the author show that it would not be feasible to detect this difference by photoelectric counting experiments. Because of the much larger Doppler broadening, even the exponential shape of the pulses is not verified in existing experiments known to us. In principle, this could be done by observing the fringe visibility curve of radiation emitted normal to a well-collimated atomic beam; but even this will not distinguish among the theories as long as the excitation is inefficient.

As Leiter suggests, we do observe that when the excitation is removed, the net radiation from many atoms decays exponentially according to Eq. (1). But this is just what NCT predicts for inefficient excitation; and a more detailed analysis[2] of the net radiation, for a given distribution of initial states, shows that NCT predicts net exponential decay with the proper time constant even for efficient excitation, if the distribution of $|a_2|^2$ is not sharply peaked.

Evidently, experiments capable of distinguishing between these theories would be possible if we could achieve high and accurately reproducible excitation. For example, suppose that by a laser pulse of controlled amplitude and duration we could pump in such a way that most of the atoms had $|a_2|^2 \geq 0.9$. QED predicts no change in the character of the emitted radiation, except for a greater intensity due to the greater pumping efficiency. NCT predicts (a) a time delay before the maximum emission is reached, which in the case of the sodium D-lines would be of the order of 100 nsec; (b) a change in the fringe visibility curve as we see more and more of the hyperbolic secant envelope. Such experiments appear feasible with presently available technology.

In summary, existing optical experiments do not permit one to decide between QED and NCT; but several new experiments capable of doing this are now feasible, two of which were just mentioned. In any event, this situation makes it clear that present experimental evidence does not establish the validity of QED, to the exclusion of alternative theories, even in the optical region.

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[1] D. Leiter, previous paper, Phys. Rev. A 1, 259 (1970).

[2] E. T. Jaynes and F. Cummings, Proc. IEEE 51, 89 (1963).

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[3]M. Duggan, thesis, Stanford University, ~~1961~~ (unpublished).

[4]J. H. Eberly, thesis, Stanford University, 1962 (unpublished).

[5]C. W. Bates, Jr., thesis, Washington University, 1968 (unpublished).

[6]M. D. Crisp, thesis, Washington University, 1968 (unpublished); M. D. Crisp and E. T. Jaynes, Phys. Rev. 179, 1253 (1969); 185, 2046 (E) (1969).

[7]C. R. Stroud, thesis, Washington University, 1969 (unpublished); C. R. Stroud and E. T. Jaynes, Phys. Rev. A 1, 106 (1970).

[8]The preliminary treatment of the Lamb shift in Ref. 6 is still based on a two-level approximation, neglecting the effect of other levels weakly excited during a transition. The result agreed with experiment in the one case (Lyman-α line), where this approximation would be expected to be good. Better calculations for other lines are underway.