Reprinted from: COHERENCE AND QUANTUM OPTICS
Edited by L. Mandel and E. Wolf
Book available from: Plenum Publishing Corporation
227 West 17th Street, New York, N. Y. 10011

SURVEY OF THE PRESENT STATUS OF NEOCLASSICAL RADIATION THEORY*

E.T. Jaynes

Washington University, St. Louis, Mo.

1. Introduction

Present quantum electrodynamics (QED) contains many very important "elements of truth", but also some clear "elements of nonsense". Because of the divergences and ambiguities, there is general agreement that a rather deep modification of the theory is needed, but in some forty years of theoretical work, nobody has seen how to disentangle the truth from the nonsense. In such a situation, one needs more experimental evidence, but during that same forty years we have found no clues from the laboratory as to what specific features of QED might be modified. Even worse, in the absence of any alternative theory whose predictions differ from those of QED in known ways, we have no criterion telling us *which* experiments would be the relevant ones to try.

It seems useful, then, to examine the various disturbing features of QED, which give rise to mathematical or conceptual difficulties, to ask whether present empirical evidence demands their presence, and to explore the consequences of modified (although

*Supported by the Air Force Office of Scientific Research, Contract No. F44620-60-0121.

This talk was scheduled as the first of a long series devoted to Quantum Electrodynamics and alternative theories. Evidently, a fully up-to-date account of the topic could not be given until the other speakers, presenting new evidence bearing on these matters, had been heard. This final version therefore differs from the talk actually given, in deletion of obsolete material and in additions to take account of work reported by others at the Conference.

perhaps rather crude and incomplete) theories in which these features are removed. Any difference between the predictions of QED and some alternative theory, corresponds to an experiment which might distinguish between them; if it appears untried but feasible, then we have the opportunity to subject QED to a new test in which we know just what to look for, and which we would be very unlikely to think of without the alternative theory. For this purpose, the alternative theory need not be worked out as completely as QED; it is sufficient if we know in what way their predictions will differ in the area of interest. Nor does the alternative theory need to be free of defects in all other respects; for if experiment should show that it contains just a single "element of truth" that is *not* in QED, then the alternative theory will have served its purpose; we would have the long-missing clue showing in what way QED must be modified, and electrodynamics (and, I suspect, much more of theoretical physics along with it) could get moving again.

That is, in a nutshell, the program I have visualized for getting my favorite subject, electrodynamics, out of the difficulties that it has been in throughout the adult life of every person here. And up to this point, I think that it is entirely non-controversial; nothing I have said thus far could offend anyone, whatever his personal views about QED. But the trouble starts when we start deciding which specific feature of QED should receive the surgical removal needed to formulate a modified theory. For, I believe our Quantum Electrodynamicists are quite correct in saying that *no* part of QED can be modified without coming into conflict with the basic Copenhagen interpretation of quantum mechanics, as applied to the electrons with which the field interacts. But the Copenhagen interpretation has become something sacred now, no longer a set of hypotheses to be tested by experiment, but an ideology which prescribes the limits of permissible thought in physics. So, no matter what aspect of QED I decide to tamper with, I can expect to be attacked by some fanatic who thinks I am committing blasphemy. And it will do no good to protest that I don't necessarily "believe" the modification; I am merely formulating a tentative hypothesis, to find out what its consequences would be; it is just the inability to comprehend that kind of subtlety that makes a fanatic. So, in spite of the apparently innocent nature of my program when stated in generalities, there is no way to carry it out explicitly without getting into controversy.

This being the case, one might as well be hung for a sheep as a lamb (no pun intended), and so, if I may compound metaphors, I decided to go straight for the jugular vein of QED, and do a surgical removal of field quantization itself. In this, I may be accused of blasphemy, but not of originality. For many physicists (among whom Planck, Schrödinger, and Franken have been quite explicit) have opined that the experimentally observed "quantum effects", such as the $E = h\nu$ relation, should be accounted for by the properties of matter and its interaction with radiation, with no need for any change in the nature of the electromagnetic field itself. From this standpoint, the troubles of QED would be regarded as symptoms that we are trying to take the same thing into account twice, and the theory I want to describe represents merely the working out of some quantitative aspects of an old, but previously not much developed, idea.

Unfortunately, it is necessary to do more than merely describe this theory. Anybody who undertakes to play this game of exploring unconventional ideas will be astonished at the kind of reactions he gets. Not that some applaud your efforts and others deplore them; that you expect, as already noted. What is astonishing is that, after the most carefully written expositions, so many on both sides insist on completely misunderstanding and misrepresenting not only your viewpoint and objectives, but also the plain documentable facts about what you have already accomplished. There is not space here enough to correct all the misinformation about Neoclassical theory to be found in the March 1972 issue of Physics Today. I cringed at the sight of that black box with its sensational tabloid headline: "The Uncertainty Principle Violated!" Let me assure you: the lady's honor is quite intact as far as I am concerned, because she has never set foot in neoclassical theory.

This extraordinary difficulty in communicating unconventional ideas - even when both parties are trying their friendly best to bring it about - means that the person exploring them must be prepared to spend a great deal of time, not on constructive things, but on clearing up past misunderstandings. So, while the primary subject of this article is the status of neoclassical theory, a secondary objective must be to try to correct a long list of almost unbelievably persistent misconceptions about what the theory is, and what we are trying to accomplish with it.

In particular, we must emphasize that (1) while this theory has already made a number of new predictions, it is still in a very incomplete, provisional state with regard to fundamentals, and just for that reason, it is still flexible and can, in many respects, still adapt itself to new facts. (2) As discussions at this conference show, it is hard to keep one's sights on the real issue. I had thought that our motivation and objectives were explained sufficiently clearly two years ago (particularly in Ref. 9, and Ref. 7, Introduction and page 110), but realized too late that I have probably contributed to a new confusion of the issue by the title of this article. So I will emphasize, to the point of belaboring it, that the issue before us is *field quantization*, whether we do or do not need it in order to account for the facts of electrodynamics. The issue is *not* the universal validity of any current form of semiclassical theory, whether concocted by me or anybody else. It is

QED, and not semiclassical theory, which has made pretensions of universal validity.

As the title indicates, we are, of course, interested in the range of validity of semiclassical theories, and in the question whether their present methods can be refined so as to enlarge their domain of validity; and not only for the reasons noted above. Independently of all deep theoretical questions, semiclassical methods have a proven usefulness in current experimental work of quantum optics, such as laser dynamics and coherent pulse propagation, which we would naturally like to increase. Furthermore, even without new experiments, every increase in the domain of applicability of semiclassical methods causes an equal and opposite decrease in the domain where QED can be claimed to be necessary, which sharpens our judgment as to where the faulty feature of QED may lie.

With all this concentration on negative aspects of QED, I will surely be accused of failing to recognize its good features. In defense, let me point out that, to the best of my knowledge, I was the first person to apply QED to problems of coherence in quantum electronics (by this, I mean real QED, and not just "Fermi Golden Rule" type approximations to it). In the middle 1950's, Professor Willis Lamb showed me his then unpublished semiclassical theory of the ammonia maser. But, having exactly the same instincts that I see in young physicists today, I felt very uneasy about the results until I could verify that they followed also from QED. To do this, it was evident that one must go beyond time-dependent perturbation theory and find ways of solving the equations of motion accurately over long times, without losing phase information.

This work, done in 1956, led to a new respect for semiclassical theories and culminated in a report that we finally got out in 1958[1], which included the beginnings of the neoclassical theory, and saw the first appearance of elliptic functions, of which one particular limiting form - the hyperbolic secant pulse - was noted (but without any comprehension of the significance this would have later, thanks to McCall and Hahn). Developments since then[2-9] are found in the Ph.D. theses of many former students, who have contributed a large mass of detailed calculations, including quite a few surprises.

2. Semiclassical Methods

Before we get into its deep philosophical meaning, let's be sure we understand what the term "Neoclassical theory" means pragmatically, by tracing its evolution from older semiclassical methods. What I will call Semiclassical A (SCA) was the original method of incorporating the electromagnetic field into quantum theory, antedating QED. SCA is what we were all taught in our first course in quantum mechanics, defined for our present purposes (which are served adequately by the model of a single nonrelativistic spinless hydrogen atom) by the Schrödinger equation

$$i\hbar\dot{\psi} = \left[\frac{(p-\frac{e}{c}A)^2}{2m} + e\phi\right]\psi \tag{1}$$

in which the electromagnetic potentials A,ϕ are considered given. This equation determines the effect of the field on the atom; from it we obtain the quantum theory of the Zeeman and Stark effects, the Einstein B-coefficients of black-body radiation theory, the Rutherford scattering law, the photoelectric cross-section, and with appropriate generalization, very much more.

SCA is incomplete in that it fails to give the effect of the atom on the field. To supply this, so that one could describe emission and scattering of radiation, there arose the "Klein Vorschrift" described in the Pauli Handbuch article[10], in which one found it necessary to make arbitrary replacements of the form $\langle F^2\rangle \to 2\langle F^+F^-\rangle$ for field quantities F, in order to obtain sensible results for rate of radiation of energy. Here F^+, F^- are the positive and negative frequency parts of F; thus the Klein Vorschrift was an ancestor of modern normal-ordering methods of QED.

Closely related to this was the "transition current method" (TCM) which is still very much in use today and is described, for example, in Schiff's textbook[11]. In TCM, one specifies initial and final states ψ_i, ψ_f for the electrons, and sandwiches an operator representing current, dipole moment, etc. between them, making the "transition current"

$$J_{fi}(x,t) = \frac{e}{mc}\ \psi_f^*\ (p - \frac{e}{c}A)\ \psi_i \tag{2}$$

or the "transition dipole moment", etc. Then we switch to classical electromagnetic theory, and calculate the fields that would be produced by such a current or dipole moment. In this way, surprisingly, we obtain the correct Einstein A-coefficients for spontaneous emission. TCM also yields many other useful results, such as the Møller e-e scattering formula.

TCM can hardly be considered as a well-motivated physical theory in its own right, because it mixes up the initial and final states in a way that defies any rational physical interpretation. Note, however, that if

$$\Psi = \sum_i a_i \psi_i \tag{3}$$

is a linear combination of stationary states, the quantity

$$J(x,t) = \frac{e}{mc} \text{ Re}[\Psi^* (p - \frac{e}{c} A)\Psi] \tag{4}$$

usually called the "probability current" will be interpreted by neoclassical theory as actual current (or, at least, its divergence will equal the divergence of the actual current). Using the expansion (3), we see that the current (4) contains all the transition currents with amplitude factors $a_i^* a_j$:

$$J(x,t) = \sum_{ij} J_{ij}(x,t)\; a_i^* a_j \tag{5}$$

Because of the above difficulty of interpretation, and because both the Klein vorschrift and TCM receive an *a posteriori* justification from QED, I would consider that they do not represent parts of any semiclassical theory, but should be regarded as convenient short-cut algorithms contained in QED.

An entirely different way of taking into account the effect of atoms on the field is based on the Ehrenfest theorem. The equations of motion for expectation values

$$\frac{\partial}{\partial t}\langle F\rangle = (i/\hbar)\langle [H,F]\rangle \tag{6}$$

resemble classical deterministic equations, and they reduce to the usual classical equation of motion for the quantity F, as the dispersion

$$(\Delta F)^2 \equiv \langle F^2\rangle - \langle F\rangle^2 \tag{7}$$

tends to zero. This was exploited in the famous 1946 paper of F. Bloch[12] on magnetic rexonance theory, resulting in a theory that I will call Semiclassical B (SCB). Imagine that we have a sample of a few milligrams of some substance containing protons; the number of them is probably of the order of $n \sim 10^{20}$, or more. The operator representing the total magnetic moment of these n protons is the sum of the individual operators:

$$M = M_1 + M_2 + \cdots + M_n \tag{8}$$

and if all spins are related to the sample and to each other in the same way, the expectations of M and M^2 are

$$\langle M\rangle = n\langle M_1\rangle \tag{9}$$

$$\langle M^2\rangle = n\langle M_1^2\rangle + n(n-1)\ \langle M_1M_2\rangle \tag{10}$$

Defining the mean square fractional fluctuations as $R \equiv (\Delta M)^2/\langle M\rangle^2$, $R_1 \equiv (\Delta M_1)^2/\langle M_1\rangle^2$, we have from (9), (10),

$$R = \frac{\langle M_1M_2\rangle - \langle M_1\rangle^2}{\langle M_1\rangle^2} + \frac{1}{n}\,\frac{\langle M_1^2\rangle - \langle M_1M_2\rangle}{\langle M_1\rangle^2} \tag{11}$$

The total moment M becomes better defined as $R\to 0$, and we see from(11) that R depends crucially on correlations between spins. We need look only at the two extreme limiting cases of (11). *Case 1*, complete positive correlation: $\langle M_1M_2\rangle = \langle M_1^2\rangle$. Then (11) reduces to $R = R_1$; the total moment of n spins is no better defined than that of a single spin. In other words, there is no "law of large numbers". *Case 2*, no correlation: $\langle M_1M_2\rangle = \langle M_1\rangle\langle M_2\rangle = \langle M_1\rangle^2$. In this case, (11) reduces to

$$R = \frac{1}{n} R_1 \tag{12}$$

and the law of large numbers is resurrected; for large n, the total moment becomes as well-defined as any classical quantity ever was.

Now, the expectation of a single moment obeys the equation of motion

$$\frac{\partial}{\partial t}\langle M_1\rangle = (i/\hbar)\langle[H,M_1]\rangle \tag{13}$$

and, from (9), we need only multiply both sides of this by n to have the equation of motion for <M(t)>. If the spins are uncorrelated (or more generally, if spin-spin correlations drop off with their separation sufficiently rapidly for an ergodic condition to hold), the relative fluctuation R will be negligible, and we have the deterministic Bloch equations for total magnetic moment of the sample. As I have shown elsewhere[13], it is such considerations that give, in large measure, the explanation for the success of statistical mechanics.

It remains to consider how the total moment $\underline{M}$ affects the radiation field. If the sample is small compared to a wavelength, it seems clear that a well-defined magnetic moment $\underline{M}$ should generate a well-defined classical electromagnetic field via the Maxwell equations

$$\nabla \times \underline{H} - \frac{1}{c}\dot{\underline{E}} = 0 \tag{14}$$

$$\nabla \times \underline{E} + \frac{1}{c}\dot{\underline{H}} = \frac{4\pi}{c}\frac{\dot{\underline{M}}}{V} \tag{15}$$

where V is the volume of the sample. In this way, for example, one finds that the open-circuit voltage induced in a coil wound in an arbitrary way about the sample, is given by $v_{OC} = \underline{H}\cdot\dot{\underline{M}}$, where $\underline{H}$ is the magnetic field at the sample due to unit current in the coil. In NMR work, the Q of the receiving circuit is usually so low that radiation damping [i.e., the effect of the field calculated from (14), (15) reacting back on the moment M] can be neglected. In high resolution NMR and ESR, it may be a complicating factor[12].

In the 1946 Bloch paper one finds the sphere representation, fast-passage solutions which include the π-pulse, etc., which prepared the way for the Hahn spin-echo experiment[14]. Further elaborations of the Bloch sphere representation led to a theoretical technique[15] for predicting complicated sequences of spin echoes, which will probably find application soon in the theory of optical pulse echoes.

The SCB method thus initiated, was applied some ten years later in the semiclassical theories of the ammonia maser, by Basov and Prokhorov[16], and by Shimoda, Wang, and Townes[17], who assumed a delta-function velocity distribution. This was generalized to a Maxwellian distribution by Lamb and Helmer[18]. Cummings and I[3], in a work devoted largely to other matters, noted that in an ammonia maser the velocity distribution will not be Maxwellian, because the electrostatic focusser is more efficient for low velocity molecules, and carried out the (by then rather trivial) generalization to an arbitrary velocity distribution. This confirmed that such experimental quantities as the starting current and frequency pulling factor are determined by the slowest few percent of the molecules, being inversely proportional to the mean square and mean cube flight times $\langle\tau^2\rangle$, $\langle\tau^3\rangle$, respectively.

From the standpoint of radiation theory, these SCB treatments of the ammonia maser differed from the Bloch magnetic resonance theory mainly in the fact that the field producing the stimulated emission was not "externally applied", but was the field previously radiated by the molecules themselves; in other words, radiation damping was now, due to the high Q of the cavity, an essential part of the theory, with the cavity Q appearing in the expressions for starting current, amplitude of oscillation, and frequency pulling factor. At this microwave frequency (24.8 GHz) ordinary (i.e., cavity-unassisted) spontaneous emission was still completely negligible, corresponding to radiative lifetimes of the order of months, while the cavity-assisted emission took place in less than a millisecond. It is true that the active sample now extended over many wavelengths, but one considered only its interaction with the TM_{01} cavity mode, whose field is constant along the length of the beam, so that again it was the total moment of all the molecules that was considered the source of a classical electromagnetic field.

The fact that the Bloch sphere representation applies equally well to any two-level system was common knowledge at Stanford University when I spent the summer there in 1947. In a sense, it was "obvious" to anyone who knew that (2x2) unitary matrices form a faithful representation of the three-dimensional rotation group; but this does not seem to have been published at the time, and it was left for Feynman, Vernon, and Hellwarth[19] to point it out in 1957, in an article where the hyperbolic secant pulse again puts in a brief appearance [loc. cit., Eq.(19)] under the heading: "Radiation Damping". As that suggests, they were considering only the back half of the pulse, where we move downward to the south pole of the Bloch sphere; but, of course, the analytical solution can be extrapolated backwards past $t = 0$, to give the rising front half of the self-induced transparency pulse. It is incredible, in retrospect, how many times this solution had been found by theoreticians before McCall and Hahn finally realized its significance. One wonders how many other important results are hiding in theoretical papers, unrecognized even by their authors, just waiting for some clever experimenter to show us what they mean.

The first application of SCB to magnetic resonance involved ordinary radio frequencies, $\sim$ 30 MHz, where "quantum effects" had always been considered so negligible as to be impossible even to detect, much less affect any physical phenomena. The step up to the ammonia maser involved a thousand-fold increase in frequency, but here again quantum effects were considered negligible for all ordinary purposes. At room temperature, for example, one had $(\hbar\omega/kT) \sim 3 \times 10^{-3}$, so that thermal noise still predominated over "quantum noise" except at extreme cryogenic temperatures, $T \leq 1°K$. Thus, also here the use of classical electromagnetic theory should not arouse any anxiety even in the most dedicated quantophile. But the next level of application of SCB, Lamb's analysis of the laser[20], represents a further 20,000-fold increase in frequency, into the region $(\hbar\omega/kT) \sim 60$, where some would expect radiation phenomena to be completely dominated by "quantum effects".

The success of Lamb's semiclassical theory in predicting a large mass of experimental facts[21] therefore came as an instructive surprise to some whose education did not include real QED, but only the standard verbal misconceptions of it (i.e., the "buckshot theory" of light, which has propagated through several generations of elementary textbooks) with which we brainwash our undergraduate students. Nearly all of them emerge from this with a mental picture according to which, as the frequency increases, the electromagnetic field gradually acquires some kind of discontinuous, granular structure which wipes out interference effects. Closely related is a persistent literal belief in that over-quoted remark of Dirac, to the effect that a given photon interferes only with itself. From the standpoint of QED such a statement is neither true nor false, but simply meaningless, for "photons" lack the individuality which the statement presupposes; there is just nothing in the mathematical formalism of QED that corresponds to any such notion as "a given photon".

The appearance of the laser as an accomplished fact struck a severe blow to these almost universally held misconceptions about quantum theory. Recall that, as recently as 1963, many physicists thought that, because of Dirac's statement, it was fundamentally impossible to observe interference between independently running lasers. And recall the uproar of 1956, when some of our best known theorists would not believe the Hanbury Brown-Twiss effect, because they thought it violated quantum theory. In both cases the experimental facts were accounted for trivially by classical electromagnetic theory, but to some quantum theorists they appeared as astonishing new phenomena, in need of deep and profound explanation. Such incidents led inevitably to new discussion about the nature of "photons", and the need for QED.

Lamb's SCB theory of the laser differed from SCB treatments of the ammonia maser in several respects. At microwave frequencies, the cavity modes are well-separated in frequency, so that emission into any mode other than the one of interest is negligible. At optical frequencies the cavity "quasimodes" are a discrete set of resonances, superimposed on a continuum of field modes like those of free space. Because of this, and the magnitude of the Einstein

A-coefficient ($\sim 10^8$ sec^{-1}), spontaneous emission into modes other than the ones of interest is hardly ever negligible, and is often the dominant physical process at work. Finally, the optical cavity is of the order of a million wavelengths long, with the normal mode field reversing sign every half-wavelength along the cavity axis. This has two consequences: (1) it is no longer the total moment of the active atoms that is the effective driving force in the classical Maxwell equations; instead, the regions of active atoms is treated as a continuous medium, with an active electric polarization density. (2) Doppler broadening, which could be neglected in the ammonia maser, is now one of the crucial things determining performance; an atom moving at thermal velocities may see up to about fifty phase reversals of the field during a radiative lifetime. By analogy with the effect of flight time in the ammonia maser[3], one would conclude that the "useful" emission must be due mostly to the few percent of active atoms that are moving nearly transversely to the axis of the optical cavity, for a mode tuned exactly on the atom's natural frequency ω_o. For the next optical mode, tuned higher by perhaps $\delta\omega = 5 \times 10^{-7}\ \omega_o$, the "useful" emission will be contributed mostly by another velocity group of atoms, namely those with $v_z \simeq \pm 5 \times 10^{-7}$ c, etc.

In spite of the important role played by spontaneous emission into continuum field modes, Lamb's theory does not consider the actual physical mechanism of this process, but instead invokes a phenomenological damping mechanism which presumably has similar effects. If we define a truncated (2x2) density matrix ρ with rows and columns referring only to the two lasing levels, its equation of motion is taken in the form

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}(\rho\Gamma + \Gamma\rho) \tag{16}$$

where the damping matrix Γ is diagonal with elements γ_a, γ_b, which are phenomenological constants interpreted as decay rates to unspecified lower levels. Thus ρ, instead of relaxing to a ground state or thermal equilibrium form, damps to zero through a kind of seepage to lower levels which are never brought explicitly into the theory.

The main effect of this is that the time of coherent interaction between any one atom and the field mode of interest is effectively limited to $T_{int} \sim (\gamma_a + \gamma_b)^{-1}$, presumably of the order of a few nanoseconds, regardless of whether the atom is emitting or absorbing. Questions of long-time coherence therefore do not arise, and Lamb takes account of the field interaction by conventional time-dependent perturbation theory carried to third order, making no use of the Bloch sphere representation.

The Lamb SCB laser theory has been described in some detail so that a few of that long list of misconceptions can be pointed out. In a later paper on QED theory of the laser, Scully and Lamb[22] give an Introduction explaining the need for QED here, by pointing out defects in the semiclassical treatment. These points are made: (1) Semiclassical theory "implies that laser radiation in an ideal steady state is absolutely monochromatic". (2) According to semiclassical theory, "oscillations will not grow spontaneously, but require an initial field from which to start". (3) "Still another problem requiring a fully quantum-mechanical theory is to determine the statistical distribution of the energy stored in the laser cavity, i.e., the 'photon' statistics".

To these assertions, we reply as follows: (1) While this criticism may apply to the Lamb Semiclassical theory as actually published, it is not in any way a limitation on that theory; Lamb could easily have calculated the linewidth of an actual laser in the steady state by taking into account the statistical fluctuations in the number of excited atoms. He simply neglected to do so. A semiclassical theory of noise in the ammonia maser (where the main source is now thermal Nyquist noise generated in the cavity walls) has stood for some time[3] as a counter-example to this claim. Perhaps one would reply that the term "ideal steady state" was intended to mean one free of number fluctuations or thermal noise. But even in such a state, so ideal as to be utterly non-physical, it is still true that each excited atom emits a wave train of finite duration, therefore finite spectral width, and therefore the total radiation will have a finite width. Again, Lamb could easily have calculated this in his semiclassical treatment if he had wished to do so. (2) As before, this is not a valid criticism of semiclassical theory *per se;* it describes only the restrictive assumptions that Lamb chose to put into his calculation. He did not get spontaneous buildup of oscillation because he assumed that every atom in the lasing levels was placed by the pumping mechanism into exactly the upper state u_a or the lower one u_b. In reality, of course, the collisional excitation mechanism will place almost every atom (i.e., all except a set of measure zero) in some linear combination $\psi(0) = c_a u_a + c_b u_b +$ (contributions from other non-lasing levels). The excited atom then has, at $t = 0$, a dipole moment proportional to $|c_a{}^* c_b|$ at the lasing frequency, and oscillations build up spontaneously as well as in QED. From a pragmatic standpoint (i.e., ignoring all philosophical differences, and looking only at the actual calculations done) the main difference between neoclassical theory and other semiclassical methods lies just in the fact that points like this are recognized and taken explicitly into account in neoclassical calculations. (3) By now, our reply can be anticipated; semiclassical theory is quite capable of giving statistical fluctuations in energy. It does this automatically when it is allowed to do so, i.e., when we refrain from putting in restrictive assumptions which amount to denying the possibility of fluctuations. One cannot justify a claim that treatment of energy fluctuations requires a "fully quantum-mechanical theory" until the predictions of both theories have been worked out and compared with experiment. This is, of course, one of the main issues here, since it involves field quantization in a very direct way. Unfortunately, at the present time neither theory has been worked out sufficiently to tell what its predictions are, and the experiments are non-existent.

To those who are surprised by this last remark, being under the impression that the theoretical situation is well understood, having been disposed of, to a large degree, already by Einstein[23] in 1909, we say that we will return to the subject of energy fluctuations in a later section, starting with the recent treatment of Scully and Sargent[24] but carrying the reasoning a few steps further. Be prepared for a bigger surprise.

The final - and by far the greatest - area of applications established for Semiclassical B theory is, of course, to the phenomena of self-induced transparency[25], resonant pulse propagation, chirping, photon echoes, etc. There is no need to go into details here, or even attempt a fair set of references, since this field is developing at such a furious rate. Many other speakers at this Conference will tell us far more about the subject than I can.

It appears that either SCB or QED can be used, almost interchangeably, to discuss the interesting subject of superradiance which, from the Program of this Conference, has now won out over the Schwarz-Hora effect for the honor of being the most discussed and least observed phenomenon in physics.

To turn to the future, many new technological possibilities await the development of reliable and continuously tunable high-power lasers. As one example, I will venture to predict that, by the time another six years have passed, the subject of third-harmonic power generation will be developing as an important side-branch of this field. When the need for it arises, you will find the necessary theory, both QED and SCB, already worked out in the thesis of Duggan[5].

In the area of quantum optics, it is clear that semiclassical theory has led to vastly more real physical predictions than QED. Indeed, the hundreds of existing experiments in this field have, with only two or three exceptions (the experiment of Clauser[26] being outstanding) been predicted and/or explained in terms of SCB theory. It is usually much simpler mathematically than QED (although there is no theorem guaranteeing this for every individual problem), and it gives a simple intuitive picture of what is happening; this is something which is not only pleasing aesthetically, and often necessary for further progress, but which is conspicuously missing in QED. On the other hand, developments in QED inspired by quantum optics (coherent state representation, etc.) have given us a number of elegant theorems. However, they remain sterile, having almost no connection with real experimental facts, and there is a history of frustration[27] in attempts to find experiments which require them. In the face of this, I am glad that I do not have to defend the claim that QED is "the only workable field theory we have".

3. Neoclassical Theory

Throughout the applications of Semiclassical B theory noted above, there was the implicit idea that neglect of field quantization was justified only because of the large number of atoms or molecules involved, and that statistical considerations like Eq.(12) would render the total moment of a sample, or the total polarization of a coherence volume, a well-defined quantity, essentially free of fluctuations, which could then serve as the source of an equally well-defined classical EM field. Although I don't think anyone ever carried out an explicit calculation along the lines of Eq.(11) to verify this, it was always assumed that there was safety in large numbers, so that the SCB calculations were not in conflict with QED, but on the contrary were good approximations to what a (usually far more complicated) QED analysis would have given.

In other words, one had the physical picture of the total moment of a large number of atoms obeying definite, deterministic equations of motion, yet one was not permitted to suppose that the moment of each individual atom behaved in that way. If you asked, "Why not?" you would get different answers from different people, but they would all involve some reference to the uncertainty principle, or complementarity, or the statistical interpretation of quantum mechanics. While no two physicists would agree on just what it was, all felt that, while it was legitimate to talk about the total moment of many atoms, some *Verbot* issued from Copenhagen prevented us from talking about the moment of a single atom in the same way.

Note that it was not just that moments of individual atoms might be unknown to you and me, i.e., that different atoms might have different moments in some statistical distribution as in classical statistical mechanics. In that case, one could say that each atom still has a definite, "objectively real" moment, but that its value was unknown, because it depends not only on the known applied field, but also on unknown microscopic details of the atom's environment. For prediction, one might then have not only a dynamical problem, but also a statistical problem to contend with. The mathematics might get quite involved, but it would remain simple and straightforward conceptually.

If the situation were as just visualized, then consideration of moments of individual atoms would amount to little more than adding statistical considerations to the previous SCB treatments to extend their range of application to the case of a few atoms, where statistical fluctuations must be taken into account, and this should not, after all, present any insurmountable difficulty. But, according to the Copenhagen interpretation, it is far worse than that; not only the numerical value, but also the very concept of the moment of an individual atom, becomes fuzzy in such a way that it is held to be physically meaningless even to ask the question, "How is the moment of an individual atom varying with time?" The reason is connected with the famous von Neumann "hidden variable" theorem[28]. Although quantum theory readily yields certain mathematical quantities $\langle F\rangle = (\psi, F\psi)$ which are called "expectation values", they are not in general expectations over any underlying ensemble, the individual members of which could be identified with the possible "true but unknown" physical situations. In magnetic resonance, for example, one can calculate the expectations $\langle M_x\rangle$, $\langle M_y\rangle$, and expectations of any functions of them: $\langle f_1(M_x,M_y)\rangle$, $\langle f_2(M_x,M_y)\rangle$, $\cdots$, etc. But since M_x and M_y do not commute, there is no underlying joint probability distribution $p(M_x,M_y)$ which yields all those "expectations" by the usual rule of probability theory

$$\langle f(M_x,M_y)\rangle = \iint p(M_x,M_y)\, f(M_x,M_y)\, dM_x\, dM_y \quad . \tag{17}$$

So, having calculated a number of expectation values, if you ask, "What is the ensemble of possible time variations for $M_x(t)$, $M_y(t)$ which would yield my calculated expectation values?", the answer is: "There is *no* such ensemble; your 'expectation values' are expectations over nothing at all. It is not only meaningless to ask what an individual moment is doing, it is even meaningless to ask for an ensemble of *possible* behaviors!"

Now in every other statistical theory ever dreamt of, if such a situation were to arise, one would recognize instantly that a logical contradiction has been found. The obvious, common-sense conclusion would be drawn that our interpretation is in error; a quantity which cannot be written as an expectation, should not be interpreted as an expectation. The mathematical quantities $\langle F\rangle$, whose usefulness is undeniable (they form the source of the radiation field in SCB) ought not to be interpreted physically as mere expectation values; they have a more substantial meaning. As many physicists, including Einstein, Schrödinger, and von Laue[29] have been pointing out for 45 years now, the Copenhagen theory slips here into mysticism; by refusing to recognize this contradiction and clinging to an unjustifiable interpretation, it ends up having to deny the existence of an underlying ensemble, and therefore, of any "objective reality" on the microscopic level.

That this denial is required by the Copenhagen interpretation, has been well recognized by Heisenberg[30], who states it many times. I give three examples: "They (i.e., opponents of the Copenhagen interpretation) would prefer to come back to the idea of an objective real world, whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them". "The ontology of materialism rested upon the illusion that the kind of existence, the direct 'actuality' of the world around us, can be extrapolated into the atomic range". "An objective description for events in space and time is possible only when we have to deal with objects or processes on a comparatively large scale, •••".

I think most physicists, even though they may profess faithful belief in the Copenhagen interpretation, still share with me a disreputable, materialistic prejudice that stones and trees cannot be either more - or less - real than the atoms of which they are composed. And, if it is meaningless to ask what an individual moment is doing, can it be any more meaningful to ask what their sum is doing?

It seems to me that the proper business of theoretical physics is to recognize these contradictions for what they are, and to try to resolve them. Instead, the Copenhagen school of thought tries to hide them from view, by proclaiming a new philosophy of human knowledge, according to which it is naive even to raise questions about "objective reality", or, for that matter, about anything that the Copenhagen theory cannot answer. Bohm and Bub[31], recognizing this, have rightly emphasized the dangers for the progress of physics in a theory which effectively contains within itself a proclamation of its own infallibility, by the device of declaring to be meaningless any question that the theory is unable to answer. For, if everyone accepted this, then even if the theory were grossly in error, the way to a better theory would be blocked; we would be prohibited from ever raising any question which might permit us to discover the errors.

For these reasons, I think that it is not only desirable, but very likely a prerequisite for any further progress in theoretical physics, that physicists insist on raising, and seeking constructive answers to, physical questions that the Copenhagen interpretation rejects as naive and meaningless, in particular, questions about the detailed mechanism by which an atom interacts with the electromagnetic field. Exactly what is happening within the atom when it is in the process of emitting or absorbing light? How do not only its dipole moment, but the entire underlying charge and current distributions, vary during the interaction? A theory which cannot answer such questions will, I think, be found inadequate to deal with the experimental facts of quantum optics before many more years have passed.

Unlike Bohm and other recent dissenters from the Copenhagen theory, however, I do not think that the way out requires anything so radical as the introduction of new "hidden variables". At least, before going that far out, let's try a more conservative treatment, and retain as much as possible of the present mathematical formalism, which, after all criticism, still contains a very large amount of truth. I have adduced reasons, highly convincing at least to me, indicating that the quantity presently called "expectation of moment" for a single atom, should not be interpreted in that manner. Nevertheless, it clearly has some kind of close connection with the physical notion of dipole moment. So let us give it a new physical interpretation which retains that connection, and see whether we get a more sensible theory which can be interpreted without mysticism. There are many possibilities to be explored here, and of course there is no guarantee that the first one we try out will prove to be the correct one. In other words, we are now at the stage of formulating tentative hypotheses about re-interpretation, not because we believe the new hypothesis is necessarily correct, but rather to find out what its consequences would be. If it proves to be unsatisfactory, then we can try out a different one. I feel strongly that, with enough persistence, this process should lead to the solution of our problems.

That is the philosophical basis for Neoclassical theory, (NCT). Mathematically, the step from SCB to NCT is so trivial as to be hardly noticeable; it consists of nothing more than taking the SCB equations already developed for the total moment of many atoms, and applying them instead to each individual atom. But conceptually, as we have just seen, this amounts to a revolutionary change in viewpoint. The proponents of the Copenhagen interpretation have ignored the dire warnings of Planck, Einstein, Schrödinger, de Broglie, von Laue, about the path they were taking; and so now we are going to ignore the dire warnings of the Copenhagen school, and proceed to do exactly what they have told us cannot be done.

Since, according to this prescription, the "expectation of moment" of an individual atom is now to be used as the source of a classical EM field, the expectation has been re-interpreted as the *actual* value of dipole moment. This amounts to a radical change in the interpretation of the Schrödinger wave function, but, following our conservative plan of action, we will not at this time attempt to postulate exactly what the new interpretation is. Instead we will be guided by the requirement of consistency with the re-interpretation of dipole moment just made, and make other changes in interpretation only to the extent demanded by consistency.

For setting forth the basic properties of NCT, we can still restrict ourselves to the simple model of a non-relativistic, spinless hydrogen atom; it turns out that the needed generalizations all go through effortlessly, in the most obvious way. We will retain all of the Schrödinger mathematical formalism associated with Eq.(1). There is the Hamiltonian

$$H = H_o + V(t) \tag{18}$$

with

$$H_o = \frac{p^2}{2m} - \frac{e^2}{r} \tag{19}$$

$$V(t) = -\frac{e}{mc}\,\underline{A}\cdot\underline{p} + \frac{e^2}{2mc^2}A^2 \quad , \tag{20}$$

the usual unperturbed stationary states $u_n(x)$:

$$H_o\,u_n(x) = E_n\,u_n = \hbar\omega_n\,u_n(x) \tag{21}$$

and the usual wave function expansion

$$\psi(x,t) = \sum_n a_n(t)\,u_n(x) \quad . \tag{22}$$

Now the net result of the long polemic just concluded is that the quantity

$$\underline{M}(t) = e\int \psi^*(x,t)\,\underline{r}\,\psi(x,t)\,d^3x = \sum_{n,k} \underline{\mu}_{nk}\,a_n^*(t)\,a_k(t) \tag{23}$$

is now taken to represent the dipole moment of the atom, where $\underline{\mu}_{nk}$ are the usual dipole moment matrix elements. From the definition of dipole moment in terms of charge density, $\underline{M} = \int \underline{r}\,\rho(x)\,d^3x$, we conclude that the charge density is given by

$$\rho(x) = e|\psi(x)|^2 \quad , \tag{24}$$

which is exactly Schrödinger's original interpretation of his wave function. According to the well-known conservation theorem which follows from the Schrodinger equation $i\hbar\dot{\psi} = H\psi$, the quantity

$$\underline{J} = \frac{e}{mc} \operatorname{Re}[\psi^* (\underline{p} - \frac{e}{c} \underline{A})\psi] \quad , \tag{25}$$

usually interpreted as (e/c) × (probability current), obeys the equation

$$\nabla \cdot \underline{J} + \frac{1}{c} \dot{\rho} = 0 \quad , \tag{26}$$

and so $\underline{J}$ may be interpreted as electric current density (in emu cm^{-2}), with the proviso that, as far as charge conservation is concerned, any other choice with the same divergence, e.g. $\underline{J}' = \underline{J} + \nabla \times \underline{Q}$, where $\underline{Q}(x)$ is any vector field, will do as well. This is one of the points of "flexibility" of NCT, that I alluded to in the Introduction; different choices of $\underline{Q}$ will alter the radiation from the atom, and at this stage we have only formal simplicity and experimental evidence to help us decide which choice is best. For the time being, we stick to the conventional choice (25).

With charge and current densities identified, we can introduce the radiation field. We use a general modal expansion: define a "cavity" by some volume V bounded by a closed surface S, and let k_λ^2, $\underline{E}_\lambda(x)$ be the eigenvalues and eigenfunctions of the boundary-value problem

$$\nabla \times \nabla \times \underline{E}_\lambda - k_\lambda^2 \underline{E}_\lambda = 0 \quad \text{in } V \quad ,$$
$$\underline{n} \times \underline{E}_\lambda = 0 \quad \text{on } S \quad , \tag{27}$$

where $\underline{n}$ is a unit vector normal to S. The resonant frequencies are $\Omega_\lambda = ck_\lambda$. The vector eigenfunctions $\underline{E}_\lambda(x)$ for which $k_\lambda \neq 0$ form, if V is simply connected, a complete set for expansion of the transverse field; if we use the Coulomb gauge, they will suffice for expansion of the vector potential in the form

$$\underline{A}(x,t) = \sqrt{4\pi}\, c \sum_\lambda Q_\lambda(t)\, \underline{E}_\lambda(x) \quad . \tag{28}$$

The magnetic field is given by

$$\underline{H}(x,t) = \sqrt{4\pi}\, c \sum_\lambda Q_\lambda(t)\, \nabla \times \underline{E}_\lambda(x) \quad , \tag{29}$$

and for the electric field we use expansion coefficients P_λ:

$$\underline{E}(x,t) = -\sqrt{4\pi} \sum_\lambda P_\lambda(t)\, \underline{E}_\lambda(x) \quad . \tag{30}$$

The Maxwell equations

$$\nabla \times \underline{E} + \frac{1}{c}\dot{\underline{H}} = 0 \tag{31a}$$

$$\nabla \times \underline{H} - \frac{1}{c}\dot{\underline{E}} = 4\pi\,\underline{J} \tag{31b}$$

then reduce, using (29), (30), to

$$\dot{Q}_\lambda = P_\lambda \tag{32a}$$

$$\dot{P}_\lambda = -\Omega_\lambda{}^2\, Q_\lambda + \sqrt{4\pi}\, c \int_V \underline{E}_\lambda(x) \cdot \underline{J}(x,t) d^3x \tag{32b}$$

respectively. The longitudinal current, being orthogonal to all the $\underline{E}_\lambda(x)$, does not contribute to the integral in (32b).

This formalism has been set up so that the total field energy is

$$H_f = \int_V \frac{E^2 + H^2}{8\pi} dV = \sum_\lambda \frac{1}{2}(P_\lambda^2 + \Omega_\lambda^2\, Q_\lambda^2) \quad , \tag{33}$$

and the Hamiltonian equations of motion based on (33) are evidently identical with the free-space Maxwell equations. To write the driven Maxwell equations (31) in Hamiltonian form, we substitute (25) and then (28) into the last term of (32b) and carry out the space integration. The driving force term of (32) then assumes the form

$$\sqrt{4\pi}\, c \int \underline{E}_\lambda \cdot \underline{J}\, d^3x = \sqrt{4\pi}\, \frac{e}{m} <\underline{E}_\lambda \cdot \underline{p}> - \frac{4\pi e^2}{m} \sum_\mu <\underline{E}_\lambda \cdot \underline{E}_\mu> Q_\mu \quad , \tag{34}$$

in which we have followed the customary notation

$$<F> \equiv \int \psi^* F \psi\, d^3x \quad , \tag{35}$$

even though these quantities no longer have the physical meaning of expectation values, but are now to be taken simply as mathematical quantities defined by (35). The "diamagnetic" term of (34), containing Q_μ, has not yet been used in any neoclassical calculation, but we carry it along in order to demonstrate the full consistency of the formalism being developed. From (34), it is evident that if we define an interaction Hamiltonian

$$H_{int} = -\sqrt{4\pi}\, \frac{e}{m} \sum_\lambda <\underline{E}_\lambda \cdot \underline{p}> Q_\lambda + \frac{4\pi e^2}{m} \sum_{\lambda\mu} \frac{1}{2} <\underline{E}_\lambda \cdot \underline{E}_\mu> Q_\lambda Q_\mu \quad , \tag{36}$$

then the Maxwell equations (31), (32) are identical with the Hamiltonian equations of motion

$$\dot{Q}_\lambda = \frac{\partial H'}{\partial P_\lambda} \tag{37a}$$

$$\dot{P}_\lambda = -\frac{\partial H'}{\partial Q_\lambda} \tag{37b}$$

with $H' = H_f + H_{int}$.

The interaction Hamiltonian (36) was constructed by the sole criterion that its negative derivative should equal (34). It is not obvious, then, whether it bears any relation to the interaction Hamiltonian denoted by V(t) in (20), and chosen by the criterion that it yield the conventional Schrödinger equation of motion for the atom. However, making use of (28), we see that (36) is equal to

$$H_{int} = -\frac{e}{mc}\langle \underline{A}\cdot\underline{p}\rangle + \frac{e^2}{2mc^2}\langle \underline{A}^2\rangle = \langle V(t)\rangle \quad . \tag{38}$$

To make this correspondence, the factor of ½ in the diamagnetic term of (36) was essential.

But (38) now enables us to carry our physical interpretation a step further. For H_{int} , from the relations (31)-(37), clearly has the physical meaning of the interaction energy between atom and field. Therefore, the quantity $\langle V(t)\rangle$, which conventional theory interprets as expectation of interaction energy, must now be re-interpreted as actual interaction energy. From (18), it then appears that the quantity $\langle H_0\rangle$, usually called the expectation of the atom's unperturbed energy, must now be interpreted instead as its actual unperturbed energy. In this manner, the requirement of consistency with our original re-interpretation of dipole moment, leads us to a fairly complete physical interpretation of the whole formalism.

It remains to put the equations of motion for the atom into a form suitable for our purposes. From (22), the Schrödinger equation (1) takes the usual form

$$i\hbar\dot{a}_n = \hbar\omega_n\, a_n + \sum_k V_{nk}(t)\, a_k \tag{39}$$

which can be written in a "quasi-Hamiltonian" form:

$$i\hbar\,\dot{a}_n = \frac{\partial H''}{\partial a_n^*} \tag{40a}$$

$$i\hbar\,\dot{a}_n^* = -\frac{\partial H''}{\partial a_n} \tag{40b}$$

with

$$H'' = \sum_n \hbar\omega_n\, a_n^*\, a_n + \sum_{nk} V_{nk}\, a_n^*\, a_k$$

$$= \langle H_o \rangle + \langle V(t) \rangle \quad . \tag{41}$$

But we can make Eqs.(40) look much more "classical" by defining real amplitudes $p_n(t)$, $q_n(t)$ as follows:

$$a_n(t) = \frac{p_n - i\omega_n\, q_n}{\sqrt{2\hbar\omega_n}} \tag{42}$$

Rewriting H" in terms of p_n, q_n, we find that the Schrödinger equation for the atom becomes

$$\dot{q}_n = \frac{\partial H''}{\partial p_n} \tag{43a}$$

$$\dot{p}_n = -\frac{\partial H''}{\partial q_n} \quad , \tag{43b}$$

where now, if we write the Hermitian matrix V_{nk} as

$$V_{nk}(t) = \hbar\sqrt{\omega_n\omega_k}\,(u_{nk} + i\, w_{nk}) \quad , \tag{44}$$

with u real and symmetric, w real and antisymmetric, H" reduces to

$$H'' = \sum_n \frac{1}{2}(p_n^2 + \omega_n^2\, q_n^2) + \frac{1}{2}\sum_{nk} u_{nk}(p_n p_k + \omega_n\omega_k q_n q_k)$$

$$+ \sum_{nk} w_{nk}\, p_n\, \omega_k\, q_k \quad , \tag{45}$$

and all imaginary quantities have disappeared.

It is only here that the full significance of the term "neo-classical" emerges. For we have created a complete classical

Hamiltonian system which yields equations of motion for both the atom and the field; at this point we can drop the primes in (37) and (43) and define a total Hamiltonian

$$H(p_n,q_n;P_\lambda;Q_\lambda) = H_{at}(p_n,q_n) + H_{int}(p_n,q_n;Q_\lambda) + H_f(P_\lambda,Q_\lambda) \tag{46}$$

where

$$H_{at} = \frac{1}{2}\sum_n (p_n^2 + \omega_n^2 q_n^2) = \langle H_o \rangle \tag{47}$$

$$H_f = \frac{1}{2}\sum_\lambda (P_\lambda^2 + \Omega_\lambda^2 Q_\lambda^2) \tag{48}$$

$$H_{int} = \text{(a quadratic form in } p_n,\ q_n\text{, with coefficients linear and quadratic in } Q_\lambda) = \langle V(t) \rangle \quad . \tag{49}$$

The resulting equations of motion (43) are identical with the conventional Schrödinger equation (1) describing how the radiation field affects the state of the atom; they are only transcribed into an unconventional notation. Likewise, the equations of motion (37) are identical with the Maxwell equations for a field driven by the transverse part of the current (25). The fact that they all turn out to be consistent (i.e., the interaction Hamiltonian which gives the correct Schrödinger equation for the atom, also gives the correct Maxwell equations for the field) is perhaps the first indication that there may be some merit in this procedure.

The dynamical variables p_n, q_n, P_λ, Q_λ are, of course, not operators but ordinary numbers as in any classical theory. The "physical quantities" are the atomic wave function and EM field vectors, ψ, E, H. Although we still use the operator $p = -i\hbar\nabla$ in the theory, as in (25), because it is a convenient and familiar notation, it no longer "represents" any particular physical quantity. In neoclassical theory, physical quantities are not represented by operators at all, any more than in classical acoustics. We are, of course, free to use operators whenever this is convenient for mathematical purposes, but whatever commutation relations they may have are simply mathematical relations that carry no physical implications of the "uncertainty principle" type.

A large class of objections to neoclassical theory that have appeared recently[24,32,33] (i.e., that its equations of motion are inconsistent, that it violates energy conservation, that it violates the uncertainty principle, etc.)arises solely from failure to comprehend the points made in the last paragraph. I hope it is now clear from the above derivations, which go into a little more detail than in our previous publications, that (1) the equations of motion for atom and field are completely consistent with each other, down to the diamagnetic term; (2) the total energy H is conserved rigorously, in consequence of the Hamiltonian form of the dynamics; (3) there is no uncertainty principle to violate. In this connection, note that the uncertainty principle that is contained in the Copenhagen theory is not an experimental fact (who has measured the dispersions of two non-commuting quantities in the same state?); it is only a limitation on that theory. Since this is easily the most obscure point in all of physics - surpassing even the second law of thermodynamics in the utter confusion with which it is presented to students - we will return to it in more detail elsewhere.

Now, referring to the program formulated in the Introduction, we have arrived at the point where a fairly definite alternative theory to QED has been constructed (although it would require much generalization before one could think of it as a complete theory of electrodynamics), and the next step is to confront it with experimental facts, to determine: (1) Just what "elements of truth" does it contain? Does it contain any that are not in QED? (2) What "elements of nonsense" are in it? Can the theory be modified to remove them?

As we see from (47), the atom in neoclassical theory is, no less than the EM field, dynamically equivalent to a set of harmonic oscillators, and so at this point you might well say, "Aha - defeat now stares you in the face; for you have dissolved everything away into nothing but classical harmonic oscillators, which will lead inevitably to the Rayleigh-Jeans law, instead of the Planck law, for black-body radiation." Not so! For this model has some tricks in it. There is another uniform integral of the motion in addition to the total energy, of a type that was never dreamt of in classical statistical mechanics, and which completely changes the laws of energy exchange between atom and field. The secret lies in the fact that the interaction Hamiltonian, in its dependence on p_n and q_n, as exhibited in (45), is quadratic rather than linear. In other words, the atom is coupled to the field not directly, but parametrically. Application of a field does not produce any force tending to displace an oscillator coordinate q_n, as we would get from a term in the Hamiltonian linear in q_n; instead, the applied field varies the "masses","spring constants", and "mutual coupling coefficients" of the atomic oscillators (47).

It is clear from (47) that the quantity

$$W_n \equiv \frac{1}{2}(p_n^2 + \omega_n^2 \, q_n^2) \tag{50}$$

is to be interpreted as the energy stored in the n'th vibration mode (formerly n'th stationary state) of the atom. The total energy of the atom, $\sum W_n$, is not a constant of the motion because of the field interaction. But one easily verifies, from (43), (45), that, thanks to the symmetry and anti-symmetry respectively of u_{nk} and w_{nk},

$$\frac{d}{dt} \sum_n \frac{W_n}{\omega_n} = 0 \quad . \tag{51}$$

In other words, we have a law of conservation of *action*. Tracing back, we find that this is the same mathematical relation that the Copenhagen theory interprets as conservation of probability: $\sum |a_n|^2$ = const. (a striking illustration of how much the "natural" physical interpretation of a formalism depends on the particular mathematical form in which it is presented), and that setting the const. equal to unity is equivalent to setting

$$\sum_n \frac{W_n}{\omega_n} = \hbar \quad . \tag{52}$$

Before discussing the physical consequences of this conservation law, let us first put the equations of motion in their most compact, easily surveyed form. Dropping the diamagnetic term, the Schrödinger equation (43) and the Maxwell equations (37) can be written respectively as

$$i\hbar \dot{a}_n = \hbar\omega_n a_n + \sum_{\lambda k} V_{nk}^{\lambda} Q_\lambda a_k \tag{53}$$

$$\ddot{Q}_\lambda + \Omega_\lambda^2 Q_\lambda = - \sum_{nk} V_{nk}^{\lambda} a_n^* a_k \quad , \tag{54}$$

where

$$V_{nk}^{\lambda} \equiv -\sqrt{4\pi} \frac{e}{m} (\underline{E}_\lambda \cdot \underline{p})_{nk} \quad . \tag{55}$$

We are here returning to the complex amplitudes $a_n(t)$ because of the familiarity and compactness of the resulting equations. However, we emphasize once again - because it is the most persistently misunderstood point in this theory - that our interpretation of $a_n(t)$ is entirely different from the conventional one. We are regarding the variables p_n, q_n as the fundamental conjugate variables of a set of harmonic oscillators comprising the atom (whose "ultimate physical nature" is a question for the future), and $a_n(t)$ as a complex variable defined by (42), representing amplitude and phase angle in the phase space of the n'th harmonic oscillator. It is so defined that its square magnitude

$$|a_n|^2 = \frac{p_n^{\ 2} + \omega_n^{\ 2}\, q_n^{\ 2}}{2\hbar\omega_n} \tag{56}$$

has now the physical meaning, not of probability, but of energy stored in the n'th mode, in units of $\hbar\omega_n$, or, what is the same thing, as action stored in that mode, in units of $\hbar$.

The action conservation law (52) - an immediate consequence of parametric coupling - has some very obvious, and very important, implications for the laws of energy exchange between field and matter. If the atom is in its n'th oscillation mode (i.e., only p_n, q_n differ from zero), then from (52) its energy is necessarily $\hbar\omega_n$, and the right-hand side of (54) vanishes (in the n'th state, the atom has no permanent dipole moment, and so $V_{nn}^{\lambda} = 0$). Therefore the atom does not excite any field oscillators.

By a suitable external perturbation A(t) it is possible to start with the atom in the n'th mode and to end with it in the m'th. The energy difference $\Delta E = \hbar(\omega_m - \omega_n)$ must then be supplied or absorbed by the radiation field. A possible way of doing this, as we know from conventional solutions of the Schrödinger equation, is to impose a weak field of frequency $(\omega_m - \omega_n)$ for a suitable length of time; this is the phenomenon of stimulated emission, or of absorption.

Suppose now that $\omega_m > \omega_n$. If both the m'th and n'th vibration modes of the atom are excited simultaneously and $V_{mn}^{\lambda} \neq 0$, the right-hand side of (54) oscillates at the difference frequency $\omega_{mn} = \omega_m - \omega_n$ (but, because of the particular way in which $p_m, q_m; p_n, q_n$ are combined in (54), not at the sum frequency). Any field oscillator for which $\Omega_\lambda \approx \omega_{mn}$ is then strongly coupled to the atom, and may be excited to a considerable amplitude. This in turn reacts back on the atom via Eq. (53), causing its state of excitation to change. In passing from mode m to mode n, the atom delivers a total amount of energy $\hbar\omega_{mn}$ to those field oscillators whose frequencies are near to ω_{mn}. Similarly, if the atom is initially excited in the n'th vibration mode, and a field oscillator of frequency $\Omega_\lambda \approx \omega_{mn}$ is originally excited with a greater energy than $\hbar\omega_{mn}$, it can deliver the energy $\hbar\omega_{mn}$ to the atom, leaving it in the m'th vibration mode.

Although the possible energies of field and atom can vary continuously in this theory, if one considers only processes in which the atom changes from one pure oscillation mode to another, the energy of field oscillators will be seen to appear and disappear in units of $\hbar\Omega$. In most experiments, these would be almost the only amounts of energy one could observe to be exchanged, because when a fraction $\alpha\hbar\Omega$ of the energy has been absorbed by an atom, it is left in a state with a large oscillating dipole moment, and continues to interact strongly with the field. Only when it has absorbed the full energy $\hbar\omega_{mn}$, or given the energy $\alpha\hbar\Omega$ back to the field, will it reach a "stationary state", where its dipole moment vanishes and the energy exchange ceases.

In the properties just noted, we see virtually all the "quantum effects" in radiation phenomena on which the early development of quantum theory was based: the Ritz combination principle, the existence of stationary (i.e., nonradiating) states of fixed energy, the interchange of energy in units of $\hbar\Omega$, absorption and induced emission. As we have shown[6,7] before, taking into account the atom's radiation reaction field (which means nothing more than finding the complete solution of (54) for all field modes and putting the result into (53)), leads to prediction of spontaneous emission with the correct Einstein A-coefficients, but a different shape (hyperbolic secant envelope) for the spontaneous emission pulse. Likewise, the beautiful experiments of R.W. Wood[34] on resonance radiation and selective excitation of atoms, are accounted for immediately.

Just before starting the mathematical development of NCT, I mentioned that we would proceed to do exactly what the Copenhagen school has told us cannot be done. That promise has now been fulfilled, for we have all been taught that the aforementioned phenomena cannot be accounted for by classical concepts at all. For example, Bohr[35] has asserted that, "Hence, in the case of atoms, we come upon a particularly glaring failure of the causal mode of description when accounting for the occurrence of radiation processes". Similar assertions are repeated endlessly throughout our textbooks.

We now see just how easy it is to do these "impossible" things; the entire secret lies in the words *parametric coupling*. The Copenhagen interpretation takes the relation $E = \hbar\omega$ as a basic postulate, and never makes any attempt to explain how or why two physical quantities so utterly different as energy and frequency should be so connected. NCT, via (52), explains this as a simple consequence of the dynamics whenever we have parametric coupling to the field, and leaves open the interesting possibility that other systems, with different kinds of field interactions, may not be subject to any such limitation.

This easy initial success of NCT has seemed to me a very powerful argument in favor of the approach used. In the action conservation law (52) we have an "element of truth" which, if not actually missing from QED, is at least present in a more physically appealing form in NCT.

The NCT formalism was set up above in some generality, in a way that emphasizes its consistency and the classical interpretability of (52) and its consequences just noted. For many applications, we can pass to the two-level approximation in which only the amplitudes $a_1(t)$, $a_2(t)$ appear, and by the dipole approximation $(\underline{E}_\lambda \cdot \underline{p})_{nk} \simeq \underline{E}_\lambda \cdot (\underline{p})_{nk}$ and a gauge transformation, the interaction term in the Schrödinger equation (but not in the Maxwell equations - this is the reason why one needs the vector potential in order to write the whole system of equations of motion in Hamiltonian form) takes the form of an electric dipole interaction. Details have been given before [3,6,7], and we recall only the result. If $\omega = \omega_2 - \omega_1 > 0$, the Schrödinger equation (39) or (53) can be written in terms of the dipole moment $M(t)$ defined by (23):

$$\ddot{M} + \omega^2 M = -(2\mu/\hbar)^2 W(t) E(t) \tag{57a}$$

$$\dot{W} = E\dot{M} \tag{57b}$$

where $E(t)$ is the electric field at the atom, and

$$W(t) \equiv \frac{\hbar\omega}{2} \left(|a_2|^2 - |a_1|^2\right) = W_1 + W_2 - \frac{1}{2}\hbar\,(\omega_1 + \omega_2) \tag{58}$$

is the energy of the atom, referred to a zero lying midway between the levels, W_n being the mode energies(50). μ is the dipole moment matrix element, denoted μ_{12} in (23). Noting the first integral of (57):

$$\dot{M}^2 + \omega^2 M^2 + (2\mu/\hbar)^2 W^2 = \omega^2 \mu^2 \tag{59}$$

which is just the action conservation law (52) in disguised form, and taking the electric field as the sum of external and radiation reaction parts:

$$E(t) = E_{ext}(t) + E_{RR}(t) \quad , \tag{60}$$

with

$$E_{ext}(t) = E_o(t) \cos[\Omega t + \theta_o(t)] \quad , \tag{61}$$

where $E_o(t)$, $\theta_o(t)$ are slowly varying, and

$$E_{RR}(t) = \frac{2}{3c^3}\dddot{M}(t) - \frac{4K}{3\pi c^3}\ddot{M}(t) \quad , \tag{62}$$

(here K is a cutoff described before[7], which is of the order of magnitude $K \sim c/a_o$ with a_o the Bohr radius; it can be calculated[6] from the detailed current distribution within the atom), we pass to the Bloch sphere representation by introducing dimensionless variables x(t), y(t), z(t):

$$\dot{M} + i\omega M = i\omega\mu(x + iy)\exp[i\Omega t + i\theta_o(t)] \tag{63}$$

$$W = \frac{1}{2}\hbar\omega\, z(t) \quad . \tag{64}$$

Thus, $\mu x(t)$, $\mu y(t)$ are respectively the components of M(t) in phase and 90° ahead of the applied field (61), while z(t) is the atom's energy, in units of $\frac{1}{2}\hbar\omega$. The first integral (59) now reduces to the equation of the unit sphere, $x^2 + y^2 + z^2 = 1$. In this representation, the Schrödinger equation describing slow changes in the energy, and magnitude and phase of the dipole moment, of the atom, has the form[7] of the "secular equations"

$$\dot{x} = \beta zx + (\alpha - \gamma z)y \tag{65a}$$

$$\dot{y} = \beta zy - (\alpha - \gamma z)x + \lambda z \tag{65b}$$

$$\dot{z} = \beta(z^2 - 1) - \lambda y \tag{65c}$$

Here

$$\beta \equiv \frac{2\mu^2\omega^3}{3\hbar c^3} = \frac{1}{2}A \; ; \qquad \gamma \equiv \frac{4K\mu^2\omega^2}{3\pi\hbar c^3} \sim 100\beta \tag{66}$$

are two constants defined by the field interaction (β is half the Einstein A-coefficient for the transition, and γ is the "dynamic Lamb shift" discussed later), while

$$\lambda(t) \equiv (\mu/\hbar)\, E_o(t) \tag{67}$$

$$\alpha(t) \equiv \Omega - \omega + \dot{\theta}_o(t) \quad , \tag{68}$$

are slowly varying measures of the amplitude and momentary frequency of the applied field. Many solutions of these equations have been given[3-7], and Equations (65), although differing in detail in the terms containing γ and β, are essentially equivalent to those of McCall and Hahn[25] in the application to intense, short pulses (i.e., $\lambda >> \beta$, $\gamma\tau << 1$). Equations (65) make a large number of detailed predictions capable of being checked by experiment, and we turn now to one case where new experimental evidence is beginning to appear.

3. Spontaneous Emission

One of the most striking differences between the predictions of QED and NCT concerns the shape of the spontaneous emission pulse from an atom. As discussed in some detail previously[6-9], NCT (via Eqs.(65) with $\lambda=0$), predicts a chirped hyperbolic secant pulse, while QED predicts the usual exponentially damped one, with the tail of the hyperbolic secant pulse $\exp(-\beta t)$ having the same decay constant as the QED pulse. This has not only stimulated some correspondence[8,9] and some experimental efforts[36], but also some more careful thought about quantum theory. This point was first raised at the Coherence Conference here six years ago, but at that time, almost every physicist whom one asked about the shape of a spontaneous emission pulse, would reply that the question was meaningless; the only observable fact is simply whether a photon has or has not been emitted, and any more detailed questions than this are forbidden because they seek to probe below the limits set by the uncertainty principle - or perhaps the Principle of Complementarity - or at least, some prohibition emanating from Copenhagen. So, we are back to that stuff again! There is just no way to avoid it. I am convinced that all fundamental questions in physics today reduce eventually to some question about the Copenhagen interpretation and the need for something better.

Nevertheless, today several laboratories are actively performing experiments to answer such questions, and in theoretical work we see graphs like the one given by Nash and Gordon[33], quite unblushingly comparing the QED and NCT emission rates of an atom as a function of its energy, varying continuously between excited and ground states. No thoughts about "instantaneous quantum jumps", no admonitions from Copenhagen about meaningless questions, impeded that work!

More seriously, let us note what experiment can tell us about the shape of a spontaneous emission pulse, because there is still some confusion about it. Suppose that this is given by the basic function

$$f(t) = \mathrm{Re}[a(t)\, e^{i\omega t}] \quad , \tag{69}$$

where $a(t)$ is a slowly varying complex envelope function. We suppose that the complication of Doppler broadening has been eliminated by placing the emitting atoms in a solid, or by observing the radiation normal to a collimated atomic beam. If the atoms emit independently (i.e., no incipient superradiance or lasing), the total electric field will be a superposition of such pulses occurring at random times t_i: $E(t) = \sum_i f(t-t_i)$. A Michelson interferometer and photocell can then measure the intensity

$$I = \overline{[E(t) + E(t-\tau)]^2} \quad , \tag{70}$$

the bar denoting a time average over a few optical cycles. Now, as we easily verify from the above relations, by observing the maximum and minimum values of I as τ varies over an optical cycle, we can determine some, but not all, details of the function $a(t)$. More specifically, this determines the convolution

$$b(\tau) = \int_{-\infty}^{\infty} a(t)\; a^*(t-\tau)dt \tag{71}$$

as follows. If the interferometer is set at a relative retardation τ, then the Michelson fringe visibility is

$$V(\tau) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{|b(\tau)|}{b(0)} \quad , \tag{72}$$

and a measurement of absolute fringe position (feasible today with a laser-calibrated interferometer) can determine the phase of $b(\tau)$.

From these relations we find the following list of pulse shapes and the corresponding fringe visibility curves:

Damped exponential [$\theta(t)$ = unit step function]:

$$a(t) = A\theta(t)\, e^{-\beta t} \qquad\qquad V(\tau) = e^{-\beta|\tau|} \tag{73}$$

Gaussian envelope:

$$a(t) = A\, e^{-qt^2} \qquad\qquad V(\tau) = \exp(-\tfrac{1}{2}q\tau^2) \tag{74}$$

Hyperbolic Secant:

$$a(t) = A\, \mathrm{sech}\, \beta t \qquad\qquad V(\tau) = \frac{\beta\tau}{\sinh \beta\tau} \tag{75}$$

Chirped Hyperbolic Secant:

$$a(t) = A[\text{sech}\beta t]^{1+i\gamma/\beta} \qquad V(\tau) = \frac{\beta|\sin\gamma\tau|}{\gamma \sinh\beta|\tau|} \tag{76}$$

The very different shapes of these visibility curves enable one to distinguish experimentally between different hypotheses concerning the envelope $a(t)$. Measurements of this type are, of course, a long since accomplished fact, and it was only a naive, but astonishingly widespread, misconception of quantum theory that led many to think they were impossible. Were it otherwise, QED would have been disproved by Albert Michelson long before any of us were born.

We leave it as an open question whether other experimental techniques (for example, observing fluctuations and coherence of resonance radiation) might enable us to measure further details of the function $a(t)$ beyond its convolution $b(\tau)$. In any event, it is clear that questions concerning the shape of the spontaneous emission wave train *are* experimentally meaningful, and that a full treatment of the effect of spontaneous emission on noise in laser amplifiers and on stability of laser oscillators will require knowledge of the correct envelope function $a(t)$. It is just a measure of how much progress in understanding we have made that, in this Conference, it never occurred to anyone to raise the kind of Copenhagen NO-NO's that were constantly in the air six years ago.

Today, the measurement, recording, and plotting of fringe visibility curves can be automated to the point where the effect on pulse shape of any change in method of excitation could be determined in a few minutes; experiments of this type could provide a wealth of data checking many details of (65) against the corresponding QED predictions.

A start on experiments of this type has been made in the interesting work of Gibbs[36], reported at this Conference. As suggested before[9], differences between QED and NCT can be seen if one can pump atoms, by laser pulses of controlled amplitude and duration, with an accurate π-pulse, whereupon the entire (or nearly the entire) hyperbolic secant envelope of (75) or (76) can be seen. With the usual inefficient pumping mechanisms the atom is excited, according to NCT, only a small distance from the South pole of the Bloch sphere (i.e., it is left nearly in the ground state, with only a small admixture of the excited state). But then we see only the exponential tail of the hyperbolic secant emission pulse, whose shape (and therefore, spectral distribution and autocorrelation function) are the same as in QED.

Gibbs has attempted more efficient pumping, and instead of analyzing the fluorescence by fringe visibility, has observed the time dependence by direct photoelectric counting. Although he claims to have disproved NCT, we believe that, in the actual conditions of the experiment, the atoms were not pumped far enough from the South pole for differences between the two theories to be observed. According to Eqs.(65), neither the frequency nor the pulse amplitude and shape were correct for pumping to the North pole. If a pulse is sufficiently intense ($\lambda >> \beta$) so that the radiation damping terms in β can be neglected during the pulse, and if the atom starts from the ground state $z = -1$ at the beginning of the pulse, then from (65) we find an integral of the motion

$$(\alpha - \gamma z)^2 = (\alpha + \gamma)^2 + 2\lambda\gamma x \quad . \tag{77}$$

The trajectory during the pulse is therefore the intersection of the parabolic cylinder (77) with the spherical surface $x^2 + y^2 + z^2 = 1$. A family of these trajectories is shown in Ref. 7, Fig. 7, for the case $\alpha = 0$; i.e., the applied field frequency Ω equals the atom's natural frequency ω. Referring to Eqs.(61)-(68), we see that Gibbs' tuning the field to the absorption line (the resonance frequency when $z = -1$), amounts instead to taking $\Omega = \omega - \gamma$, lower by the "dynamic Lamb shift", or $\alpha = -\gamma$. But then, according to (77), the North pole cannot be reached. The maximum attainable value of z is found by setting $x^2 = 1 - z^2$ in (77) and solving the resulting cubic equation. We find

$$1 + z_{max} = \frac{2a}{\sqrt{3}} \sinh\left[\frac{1}{3} \sinh^{-1}\left(\frac{\sqrt{27}}{a}\right)\right] \quad , \tag{78}$$

with

$$a \equiv \frac{2\lambda}{\gamma} = \frac{E}{E_{crit}} \quad , \tag{79}$$

where E_{crit} is the critical field at which the trajectory just reaches the equator, $z_{max} = 0$, on the Bloch sphere, before turning downward again. According to (78), in order to pump anywhere near the North pole, requires $a > 10$, which by our estimates is far greater than reached in the Gibbs experiment. Furthermore, (78) is only an approximation which neglects the spontaneous emission terms proportional to β in (65), and which would cause a further southward drift. An estimate based on Eq.(3.28) of Ref. 7 gives for the amount of this drift in latitude, during a pulse of length t,

$$\delta\theta \simeq \beta \int_0^t \sqrt{1 - z^2}\, dt \quad , \tag{80}$$

which, for the long-tailed pulse realized in Gibbs' experiment, could amount to about one-quarter radian. Finally, any chirp in the pumping pulse will cause a further decrease in z_{max}.

A full analysis of this experiment will require considerable time and will be reported elsewhere. For the time being, we note that, if the dynamic Lamb shift (i.e., terms proportional to γ in (65), which give rise to the chirping of the pulse (76)) is a real effect, then pumping to the North pole could not have been achieved in the Gibbs experiment unless the laser were retuned to $\Omega=\omega$, the only case where the trajectory (77) can reach the point $z = +1$. In this case, we believe the experiment failed to reach the conditions where differences in the theories could be seen. On the other hand, if the dynamic Lamb shift does not exist, then the Gibbs experiment was, in all probability, a valid disproof of Eqs.(65). One would then investigate whether a different choice of the vector Q discussed following Eq.(26) might give a spontaneous emission law in agreement with the experiment, or whether some other aspect of NCT could be modified.

In any event, two conclusions are: (1) We should have more experiments of this type, with different laser tunings tried out, and with cleaner pulse shapes, in order to make a clear-cut decision. It is, of course, too much to expect that the first experiment tried in a new field will settle all questions; it serves rather to indicate what the real technical problems are in making a decisive experiment. (2) The issue of the reality of the dynamic Lamb shift chirp appears a very crucial one.

In this latter connection, the relation between the two theories is brought out in a beautiful way in the work of Ackerhalt, Eberly, and Knight[37] reported at this Conference. In pseudospin notation, the equations of motion of a two-level atom interacting with its self-made field can be written in a common form. σ_3 is the energy of the atom, in units of $\frac{1}{2}\hbar\omega$, and $\sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2)$ corresponds to the rotating moment component $\mu^{-1}(M + \dot{M}/i\omega)$. In QED the σ's are operators; in NCT they are numbers, the same numbers that would in QED be called the expectation values of those operators. In either case the equations of motion of a decaying atom take the form

$$\dot{\sigma}_- = [-i(\omega_0+\gamma\sigma_3) + \beta\sigma_3]\sigma_- \tag{81}$$

$$\dot{\sigma}_3 = -4\beta\, \sigma_+ \, \sigma_- \tag{82}$$

in which β and γ are the same radiation constants defined in (66), γ is identical with the "Crisp shift" Δ_c of Ackerhalt, Eberly and Knight (AEK). In this form the chirping is evident, since the oscillation frequency is $(\omega+\gamma)$ when $\sigma_3 = 1$ (upper state), and $(\omega-\gamma)$ when $\sigma_3 = -1$ (ground state). Indeed, if we make the substitutions

$z = \sigma_3$, $x - iy = \sigma_{-}\exp(i\omega t)$, Eqs.(81) and (82) become identical with (65) with $\lambda=\alpha=0$, and have solutions

$$z(t) = -\tanh\beta(t-t_o) = \gamma^{-1}\delta\omega(t) \tag{83}$$

$$[x(t) - iy(t)] = [x(t_o) - iy(t_o)]\,\mathrm{sech}\,\beta(t-t_o)\,\exp[-i\theta(t)] \tag{84}$$

$$\theta(t) = \int_{t_o}^{t}\delta\omega(t)dt = (\gamma/\beta)\log\mathrm{sech}\,\beta(t-t_o) \quad , \tag{85}$$

where t_o is the time of maximum emission, when the trajectory crosses the equator on the Bloch sphere. The NCT equations thus predict the chirped hyperbolic secant pulse (76).

But now watch closely at how these effects are wiped out by the magic of QED. Starting from the same equations of motion (81), (82), we now have the operator identities

$$\sigma_3\sigma_{-} = -\sigma_{-} \tag{86}$$

$$2\sigma_{+}\sigma_{-} = 1 + \sigma_3 \quad , \tag{87}$$

as a result of which the coupled nonlinear equations (81), (82) collapse to uncoupled linear ones

$$\dot{\sigma}_{-} = -[i(\omega_o-\gamma)+\beta]\sigma_{-} \tag{88}$$

$$\dot{\sigma}_3 = -2\beta(1+\sigma_3) \quad , \tag{89}$$

with the solutions

$$\sigma_{-}(t) = \sigma_{-}(0)\ e^{-\beta t}\ e^{-i(\omega_o-\gamma)t} \tag{90}$$

$$[1+\sigma_3(t)] = [1+\sigma_3(0)]\ e^{-2\beta t} \quad , \tag{91}$$

and now we are back to ordinary garden-variety exponential decay with no chirp! This little example is worthy of deep contemplation.

The prediction of chirp in one theory and not in the other, would appear to be a gross qualitative difference between them, which should be easily accessible to experimental check. One would think that there must surely be some experiment already done, which could settle the question whether this chirp does or does not exist. However, such experiments are surprisingly difficult to find. In the hope of inspiring someone else to invent an experiment, let us note that the easiest thing to see is probably the greater spectral width caused by chirp, if we have efficient, North-pole pumping so that the entire pulse (76) is seen. The exact spectrum of this chirped pulse was given in Ref. 7, Eq.(4.16) with some factors of π omitted. To correct this, and indicate how more general spectra might be calculated, we start from the basic Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} \operatorname{sech}\beta t \, \exp\{i[(\omega-\omega_0)t - \theta(t)]\}dt$$

$$= 2^{i(\gamma/\beta)} \frac{\Gamma[\tfrac{1}{2}+\tfrac{1}{2}i\beta^{-1}(\omega-\omega_0+\gamma)]\Gamma[\tfrac{1}{2}-\tfrac{1}{2}i\beta^{-1}(\omega-\omega_0-\gamma)]}{\beta\Gamma\,(1+i\beta^{-1}\gamma)} \quad , \tag{92}$$

for whose evaluation we are indebted to L.P. Benofy. Then, thanks to the identities

$$|\Gamma(1 + iy)|^2 = \pi y \operatorname{cosech}\pi y \quad ,$$

$$|\Gamma(\tfrac{1}{2} + iy)|^2 = \pi \operatorname{sech}\pi y \quad ,$$

we find for the spectral density, normalized to

$$\int_0^{\infty} I(\omega)d\omega = 1 \quad ,$$

$$I(\omega) = \frac{\beta}{4\pi}|F(\omega)|^2 = \frac{1}{2\gamma}\frac{\sinh(\pi\gamma/\beta)}{\cosh(\pi\gamma/\beta) + \cosh[\pi(\omega-\omega_0)/\beta]} \quad . \tag{93}$$

This is essentially flat in the interval $(\omega_0-\gamma)<\omega<(\omega_0+\gamma)$, and zero outside that range, with rapid but smooth transition regions, in which $I(\omega)$ rises from 4% to 96% of its maximum value in an interval $\delta\omega=2\beta$. Thus, the full width of a spontaneous emission line should be about twice the Lamb shift of that line, which is typically of the order of 100 times the "natural line width" of conventional theory. But if we have inefficient excitation, we have an essentially flat-topped spectrum in the narrower width $(\omega_0-\gamma)<\omega<(\omega_0+\gamma z_{max})$, whose upper limit varies with the efficiency of excitation. Hopefully, someone will think of a simple experiment which could confirm or refute this prediction.

5. Field Fluctuations

Another area where it has been claimed that classical electromagnetic theory is inadequate, concerns random fluctuations of fields. These are of two kinds: the thermal fluctuations which we observe in the laboratory as Black Body radiation or Nyquist noise in electrical circuits, and the "zero-point" energy or "vacuum fluctuations" arising from field quantization, whose reality is one of the points at issue here. On the one hand, we have been told here that vacuum fluctuations are "very real things", and that they are the physical cause of spontaneous emission and the Bethe logarithm part of the Lamb shift. Such ideas lead to simple contradictions, which have never been adequately covered up in QED.

For example, if it is true that the Einstein A-coefficients are due physically to zero-point fluctuations of the field, then why is it that the derivation of the Planck law based on the A and B coefficients, leads[33] to a result that does *not* include the zero-point energy? It seems to me that we have here either a flagrant logical contradiction, an error in calculation, or an incorrect interpretation, quite likely all three. For the conventional "derivation" of the Planck law will not bear inspection; we calculate the A-coefficient as if an atom were emitting into field-free space instead of into thermal radiation, and the B-coefficients as if the spontaneous emission were turned off. In reality, these effects interfere with each other[38] in a way that is certainly not negligible. According to either QED or NCT the conventional A - and - B-coefficient argument is just too crude to deal with the problem.

If we were willing to use the same standards of logic as those who accept the conventional derivation, we could claim to have derived the Planck law from NCT; for we too can derive the conventional A and B coefficients. But a valid derivation must obtain the spectral distribution from the full dynamics without making either "Fermi Golden Rule" type approximations or inadmissible physical assumptions about independence of spontaneous and induced processes. At the present time, neither QED nor NCT has produced any respectable derivation of the Planck law. (Of course, its derivation from the canonical ensemble of quantum statistical mechanics is trivial, but the problem here is to produce the detailed physical mechanism by which that distribution is brought about, a problem beyond the scope of equilibrium statistical mechanics).

To those who believe that zero-point fluctuations are the physical cause of the main part of the Lamb shift, they must then be "very real things" at least up to the Compton cutoff frequency, $\hbar\omega = mc^2$, to get the right Bethe logarithm. Please calculate the *numerical value* of the resulting energy density in space, the turbulent power flow from the corresponding Poynting vector, etc., and then tell us whether you still believe the zero-point fluctuations are physically real (for the Poynting vector, we get 6×10^{20} megawatts cm^{-2}; the total power output of the sun is about 2×10^{20} megawatts; real radiation of that intensity would do a little more than just shift the 2S level by 4 microvolts).

Now let us examine a rather milder problem, the energy fluctuations in thermal radiation. According to conventional QED treatments[24], the mean square energy fluctuation of the cavity modes in bandwidth $d\Omega$ is

$$(\Delta E)^2 = (\hbar\Omega)^2[\langle n^2\rangle - \langle n\rangle^2]g(\Omega)\, d\Omega \quad , \tag{94}$$

where $g(\Omega) = \Omega^2 V/\pi^2 c^3$ is the mode density function, and n is the number of photons in a single mode (assumed to have the same probability distribution for all modes in the small frequency interval $d\Omega$). For any field mode in a state describable by the $P(\alpha)$ distribution, we readily find a generalization of Einstein's formula [see Ref. 24, Eq.(21)]:

$$\langle n^2\rangle - \langle n\rangle^2 = [\langle|\alpha|^4\rangle - \langle|\alpha|^2\rangle^2] + \langle n\rangle \quad , \tag{95a}$$

in which the term in square brackets represents the mean-square fluctuations to be expected if α were a classical field variable, while the additive term $\langle n\rangle$ arises solely from field quantization, and was interpreted by Einstein in terms of particles.

If $P(\alpha)$ is Gaussian: $P(\alpha) = (\pi\langle n\rangle)^{-1} \exp(-|\alpha|^2/\langle n\rangle)$, the "classical" contribution reduces simply to $\langle n\rangle^2$, and (95a) becomes

$$\langle n^2\rangle - \langle n\rangle^2 = \langle n\rangle^2 + \langle n\rangle \quad . \tag{95b}$$

In this case, (94) can be written in the suggestive form given by Einstein:

$$(\Delta E)^2 = \frac{1}{N} <E>^2 + \hbar\Omega <E> \quad , \tag{96}$$

where $<E> = \hbar\Omega <n> g(\Omega)d\Omega$ is the average energy in the range $d\Omega$, and $N = g(\Omega)d\Omega$ is the number of modes considered. While we have no direct experimental confirmation of this formula, there is at least one case where there are independent theoretical grounds for supposing it is correct. Note that, up to this point, $<n>$ can have any frequency dependence. In the case of thermal equilibrium, it is given by the Planck law: $<n> = [\exp(\hbar\Omega/kT)-1]^{-1}$, from which we find

$$\frac{\partial <n>}{\partial T} = \frac{\hbar\Omega}{kT^2} (<n>^2 + <n>) \quad , \tag{97}$$

and (96) then reduces to a general theorem of statistical mechanics, relating the energy fluctuations of any thermodynamic system at constant volume to its heat capacity:

$$(\Delta E)^2 = kT^2 C_v \quad . \tag{98}$$

Because of the generality of (98) - it holds equally well in classical or quantum statistical mechanics, whenever we represent thermal equilibrium by the canonical ensemble - there is a strong presumption that (98) is a universally valid relation, quite independently of its above derivation from QED. And in turn, (98) is only a special case of a far more general relation giving fluctuations and covariances of any physical quantities, over any probability distribution derivable from the principle of maximum entropy[39]. So we will accept Einstein's relation (96) for thermal radiation.

How, then, is semiclassical theory to account for the "field quantization" term $\hbar\Omega <E>$ of Einstein's relation? To answer this, note that our starting equation (94) presupposed that different mode amplitudes were statistically independent, so that one could simply add up the mean-square fluctuations of the different modes, without any cross-product terms. But a moment's thought about the physical mechanism by which thermal equilibrium is maintained, shows that this cannot be correct. Each elementary emission or absorption process, exchanging an amount of energy $\hbar\Omega$ if it goes to completion, does not do so with just a single mode; it must affect simultaneously the amplitudes of many modes lying in a frequency band $\delta\Omega \sim t^{-1}$, where t is the duration of the process. We have, therefore, a non-zero correlation between the energies stored in two modes, if their frequencies are sufficiently close so that both modes "see" the same elementary emission or absorption process.

Now we have seen already in (11) how much small correlations can affect fluctuations when we are adding up a large number of small terms. The slightest positive correlation in the moments of individual spins was enough to abrogate the usual "law of large numbers". We are now faced with exactly the same phenomenon; although correlations between any two modes are extremely small, and could surely be neglected if we were considering only a few modes, the point is that they are systematic, tending in the same direction for every emission or absorption process, and every pair of modes, and their number grows like N^2, while the number of terms taken into account in (94) is only N. To estimate N: if $V = 10\ cm^3$ and $d\Omega = 10^{-4}\Omega$, then at infrared frequencies where this treatment is relevant, we have $N \simeq 10^7$. Therefore, if intermode correlations $\langle nn' \rangle - \langle n \rangle\langle n' \rangle$ were as large as a millionth of the mode variances $\langle n^2 \rangle - \langle n \rangle^2$, their total contribution might be larger than (94). Obviously, then, before we can make any pretense of having an honest calculation, we must go back to (94) and restore the missing terms.

Let the k'th mode have resonant frequency ω_k. On either classical or quantum electromagnetic theory, we may write the energy stored in this mode as $E_k = \hbar\omega_k n_k$; in classical theory, the number n_k thus defined is a continuously variable positive real number, while in QED it is an operator with non-negative integer eigenvalues. With this notation, the beginnings of the derivation involve the same formal equations in either theory. The full expression for the energy fluctuation is then

$$(\Delta E)^2 = (\hbar\Omega)^2 \sum_{k,r} [\langle n_k n_r \rangle - \langle n_k \rangle\langle n_r \rangle] \quad , \tag{99}$$

in which we sum over all modes whose frequencies are in a narrow range $d\Omega$ about Ω. We choose $d\Omega$ small enough so that we may replace all ω_k by Ω, and large enough so that $N \gg 1$; as noted above, this is no real limitation. We also suppose, as before, that the variation of $\langle n_k \rangle$ over this small frequency interval is negligible, so that we may set all $\langle n_k \rangle = \langle n \rangle$. With these understandings, (94) is seen as an approximation to (99) which retains only the diagonal terms $r = k$.

Einstein's relation (96) may also be written in the form

$$(\Delta E)^2 = (\hbar\Omega)^2 \sum_{k} [\langle n_k \rangle^2 + \langle n_k \rangle] \quad . \tag{100}$$

Now the correlation $\langle n_k n_r \rangle - \langle n_k \rangle\langle n_r \rangle$ will be appreciable when $|\omega_k - \omega_r| \lesssim \delta\Omega \sim t^{-1}$, the spectral width of the emission or absorption process, and negligible otherwise. Therefore, if $d\Omega < \delta\Omega$, (99) and (100) cannot be equal independently of our choice of $d\Omega$; for one is proportional to $d\Omega$, the other to $(d\Omega)^2$. But if $d\Omega >> \delta\Omega$, the general condition for (99), (100) to be equal independently of our choice of $d\Omega$, is

$$\sum_r [\langle n_k n_r \rangle - \langle n_k \rangle \langle n_r \rangle] = \langle n_k \rangle^2 + \langle n_k \rangle \quad , \tag{101}$$

in which we may now sum over all modes r, since contributions outside the range $|\omega_k - \omega_r| \lesssim \delta\Omega$ are negligible.

With the result (101) it is now apparent why neither QED nor semiclassical theory has yet produced any adequate treatment of this problem. For, according to (95b), QED achieves equality in (101) by using only the diagonal term $r = k$, and ignoring the others. Evidently, then, a further calculation is needed, to show that

$$\sum_r{}' [\langle n_k n_r \rangle - \langle n_k \rangle \langle n_r \rangle] = 0 \quad , \tag{102}$$

the prime denoting that the term $r = k$ is deleted. If (102) does not hold, then a correct QED calculation will not lead to the Einstein relation after all. Evidently, a theory which gives a right answer from a demonstrably bad approximation, is thereby in just as much trouble as if it had given a wrong answer from a good calculation.

In classical EM theory, with a gaussian field distribution (or very nearly so, i.e. gaussian but for these small correlations, which are not necessarily described by a multivariate gaussian distribution), the marginal probability distribution of each n_k will still be, to very great accuracy, of the Boltzmann exponential form: $P(n)\,dn = \langle n \rangle^{-1} \exp(-n/\langle n \rangle) dn$. The diagonal term of (101) is then just $\langle n_k \rangle^2$, and so semiclassical theory will have accounted for the Einstein relation if it can be shown that, in contrast to (102),

$$\sum_r{}' [\langle n_k n_r \rangle - \langle n_k \rangle \langle n_r \rangle] = \langle n_k \rangle \quad . \tag{103}$$

Until calculations to check (102), (103) have been carried out, both theories leave us in just the same state of uncertainty, for the same reason. Merely to exhibit the unsolved problems in this symmetrical way, shows how unjustified it is, in our present state of knowledge, to claim that QED is right and classical EM theory wrong, in the matter of energy fluctuations. But let us try to understand the situation a little better, by estimating the magnitude of these correlations.

Consider first semiclassical theory. The following argument makes no pretense of being a rigorous derivation, because it does not go into details of the matter-field interaction; however, it gives us an order-of-magnitude estimate very easily. Suppose that an elementary emission process at frequency Ω produces an increment $\delta n_k \ll 1$, which decays (through absorption processes not analyzed in detail here) with a characteristic lifetime τ, and this occurs, on the average, m times per second. The present value of n_k will be the result of past emissions, and its average is

$$\langle n_k \rangle = m\tau \ \delta n_k \tag{104}$$

since there were, on the average, $m\tau$ such increments during one lifetime τ in the immediate past. Now note that

$$\langle n_k n_r \rangle - \langle n_k \rangle \langle n_r \rangle = \langle (n_k - \langle n_k \rangle)(n_r - \langle n_r \rangle) \rangle \quad . \tag{105}$$

An elementary emission process increases $(n_k - \langle n_k \rangle)(n_r - \langle n_r \rangle)$, on the average, by $\delta n_k \ \delta n_r$; and this also persists for a time of the order τ, so the present value of (105) will be likewise the resultant of emissions over a time τ in the past:

$$\langle n_k n_r \rangle - \langle n_k \rangle \langle n_r \rangle = m\tau \ \delta n_k \ \delta n_r = \langle n_k \rangle \delta n_r \quad . \tag{106}$$

But the total energy emitted in the elementary process is $\sum \hbar\omega_r \ \delta n_r = \hbar\Omega$, or, since all $\omega_r \simeq \Omega$,

$$\sum_r \delta n_r = 1. \tag{107}$$

And so, summing (106) over all $r \neq k$, we have just Eq.(103)!

In QED, the situation is even more interesting. Let $\psi_m (m = 1,2)$ be the ground and excited states of an atom, ϕ_o the state of the field with n_1 photons in mode 1, n_2 in mode 2, etc., and ϕ_k the field state which differs from ϕ_o only in that one more photon is in mode k, while ϕ_{-k} is the field state differing from ϕ_o only in that one photon has been removed from mode k. If the atom is initially in its excited state, $\Psi(0) = \psi_2\phi_o$, then at a later time the state vector will be very accurately (i.e., retaining all terms that can grow secularly in first order),

$$\Psi(t) = a(t)\ \psi_2\phi_o + \sum_k b_k(t)\ \psi_1\phi_k \quad , \tag{108}$$

but in this state, we find

$$\langle n_r\rangle = n_r + |b_r|^2 \tag{109}$$

$$\langle n_k n_r\rangle = n_k n_r + n_k|b_r|^2 + n_r|b_k|^2 \quad , \tag{110}$$

and the correlation is negative:

$$\langle n_k n_r\rangle - \langle n_k\rangle\langle n_r\rangle = -|b_k|^2\ |b_r|^2 \quad . \tag{111}$$

Evidently, from (109), the probability $|b_k|^2$ now plays the role of δn_r in the classical derivation. When the atom has reached its ground state,

$$\sum_r |b_r|^2 = 1 \quad , \tag{112}$$

which corresponds to (107). If we suppose that this emission process takes place m times per second, and the field relaxes back with a lifetime τ, so that (104) still holds, the equilibrium value of the correlation will be $m\tau$ times (111), and summing over r yields, instead of (102),

$$\sum_r{}' [\langle n_k n_r\rangle - \langle n_k\rangle\langle n_r\rangle] = -\langle n_k\rangle \quad , \tag{113}$$

which just cancels out Einstein's "particle" term! Likewise, we could analyze absorption processes. Starting with the atom in its ground state, $\Psi(0) = \psi_1\phi_o$, we have at time t just (108) with the subscripts 1,2 interchanged, and ϕ_k replaced by ϕ_{-k}. Equations (109) and (110) still hold, but with negative signs for the three terms containing $|b|^2$. This leads back to (111) without modification. Thus, starting from a field state ϕ_0 without correlations, either an emission or absorption leaves it in a new state with negative correlations given by (111).

Evidently, much better calculations to check (102) in QED and (103) in classical theory, are needed before this issue can be finally resolved. We have, however, some grounds for thinking that the situation may be exactly the reverse of what we have all been taught; i.e., the fluctuation term $\hbar\Omega\langle E\rangle$ in (96), which Einstein interpreted as giving the radiation field a "particle" aspect, is accounted for after all by classical EM theory, as the effect of small intermode correlations that Einstein and all subsequent writers except von Laue[40] seem to have neglected. But in QED, the correlations are negative, canceling out the field quantization contribution, and so QED fails to give the presumably correct Einstein fluctuation law.

6. Conclusion

We have not commented on the beautiful experiment reported here by Clauser[26] which opens up an entirely new area of fundamental importance to the issues facing us. The situation is, in fact, so new that it will require much analysis, based on greater knowledge of the exact experimental conditions, before we will be in a position to make any constructive comments beyond the obvious suggestion that the experiment should be repeated with circular polarization. The implications of Bell's theorem[28], as applied to this experiment, are so astonishing that it will require much deep contemplation to digest and understand it.

What it seems to boil down to, is this: a perfectly harmless looking experimental fact (nonoccurrence of coincidences at 90°) which amounts to determining a single experimental point - and with a statistical measurement of unimpressive statistical accuracy - can, at a single stroke, throw out a whole infinite class of alternative theories of electrodynamics, namely all local causal theories. The mind boggles at the thought that any such thing could be possible. I think everybody's first impression is that there must be something wrong in any argument that purports to draw conclusions of such sweeping generality from practically no premises.

At the present time, all I can say is that to date I have not been able to find any flaw in the mathematics or logic, and to the best of my knowledge, nobody else has claimed to do so. Obviously, this argument deserves, and will receive, the closest scrutiny the human mind is capable of bringing to bear on it. If it survives that scrutiny, and if the experimental result is confirmed by others, then this will surely go down as one of the most incredible intellectual achievements in the history of science, and my own work will lie in ruins. I wish John von Neumann were here to see it.

References

1. E.T. Jaynes, Stanford Microwave Laboratory Report #502 (May 1958).
2. E.T. Jaynes in *Quantum Electronics*, ed. C.H. Townes, (Columbia University Press, New York, 1960) p. 287.
3. E.T. Jaynes and F.W. Cummings, Proc. IEEE *51*, 89 (1963).
4. J.H. Eberly, Ph.D. Thesis, Stanford University (1962).
5. M.J. Duggan, Ph.D. Thesis, Stanford University (1963).
6. M.D. Crisp and E.T. Jaynes, Phys. Rev. *179*, 1253 (1969).
7. C.R. Stroud and E.T. Jaynes, Phys. Rev. A *1*, 106 (1970). Note that, in the transition from galley proof to page layout, pages 118 and 119 became scrambled. To make sense, the text should be read in the following sequence:

Page	*Column*	*Lines*
118	1	1 - 3
118	2	4 - end
119	1	1 - 5
118	1	4 - end
118	2	1 - 3
119	1	6 - end

8. D. Leiter, Phys. Rev. A *2*, 259 (1970).
9. E.T. Jaynes, Phys. Rev. A *2*, 260 (1970).
10. W. Pauli, *Die allgemeinen Prinzipien der Wellenmechanik*, Handb. d. Phys. 2. Aufl. Band *24*, 1. Teil (1932), p. 204. Reprinted by Edwards Brothers Inc., Ann Arbor, Mich. (1946).
11. L.I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Co., Inc., New York, 1949) p. 255.
12. F. Bloch, Phys. Rev. *70*, 460 (1946); see also Rabi, Ramsey and Schwinger, Revs. Mod. Phys. *26*, 167 (1954). For a discussion of radiation damping in these experiments, see Bruce, Norberg, and Pake, Phys. Rev. *104*, 419 (1956); S. Bloom, J. Appl. Phys. *28*, 800 (1957).
13. E.T. Jaynes, Phys. Rev. *108*, 171 (1957), particularly Sec. 18.

14. E.L. Hahn, Phys. Rev. *80*, 580 (1950).

15. E.T. Jaynes and A.L. Bloom, Phys. Rev. *98*, 1099, 1104 (1955).

16. N.G. Basov and A.M. Prokhorov, J. Exp. Theor. Phys. USSR, *27*, 431 (1954); *28*, 249 (1955).

17. K. Shimoda, T.C. Wang, and C.H. Townes, Phys. Rev. *102*, 1308 (1956).

18. W.E. Lamb and J.C. Helmer, Stanford Microwave Laboratory Report #311 (1956); J. Appl. Phys. *28*, 212 (1957).

19. R.P. Feynman, F.L. Vernon, and R.W. Hellwarth, J. Appl. Phys. *28*, 49 (1957). Their radiation damping results, with the inevitable hyperbolic secant, had been found also by S. Bloom, J. Appl. Phys. *27*, 785 (1956).

20. W.E. Lamb, Jr., Phys. Rev. *134*, A1429 (1964).

21. A. Szöke and A. Javan, Phys. Rev. Letters *10*, 521 (1963); R.L. Fork and M.A. Pollack, Phys. Rev. *139*, A1408 (1965); B. Pariser and T.C. Marshall, Appl. Phys. Letters *6*, 232 (1965).

22. M.O. Scully and W.E. Lamb, Jr., Phys. Rev. *159*, 208 (1967).

23. A. Einstein, Phys. Zeit. *10*, 185, 323, 817 (1909).

24. M.O. Scully and M. Sargent III, "Physics Today" (March 1972) p. 38. See also the extensive discussion in S. Tomonaga, *Quantum Mechanics* (North-Holland Publ. Co., Amsterdam, 1962) Ch. 2.

25. S.L. McCall and E.L. Hahn, Phys. Rev. Letters *18*, 908 (1967), Phys. Rev. *183*, 457 (1968).

26. J.F. Clauser, "Experimental Limitations to the Validity of Semi-classical Radiation Theories", this volume, p. 111; see also Phys. Rev. Letters *23*, 880 (1969); *28*, 938 (1972).

27. T.L. Paoli, Phys. Rev. *163*, 1348 (1967).

28. J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, N.J., 1955). For newer developments, see J.S. Bell, Rev. Mod. Phys. *38*, 447 (1966); E.P. Wigner, Am. J. Phys. *38*, 1005 (1970); L.E. Ballentine, Rev. Mod. Phys. *42*, 358 (1970).

29. W. Heisenberg, in *Neils Bohr and the Development of Physics*, ed. W. Pauli (Pergamon Press, New York, 1955) p. 24.

30. W. Heisenberg, *Physics and Philosophy* (Harper and Bros. Publishers, New York, 1958). The above quotations are found on pp. 129, 145, 164.

31. D. Bohm and J. Bub, Rev. Mod. Phys. *38*, 453 (1966).

32. I.R. Senitzky, Phys. Rev. Letters *20*, 1062, 1277 (1968).

33. F.R. Nash and J.P. Gordon, "The Implications of Radiative Equilibrium in Jaynes' Extension of Semiclassical Radiation Theory", presented at this Conference, p. 623.

34. R.W. Wood, *Physical Optics* (Macmillan Co., New York, 1934).

35. N. Bohr, *Atomic Theory and the Description of Nature* (Cambridge University Press, 1934); reprinted in 1961; p. 13. Similar remarks are found on pp. 32, 80, 108.

36. H.M. Gibbs, "A Test of Jaynes' Neoclassical Theory: Incoherent Resonance Fluorescence from a Coherently Excited State", presented at this Conference, p. 83.

37. J.R. Ackerhalt, J.H. Eberly, and P.L. Knight, "A Quantum Electrodynamic Investigation of the Jaynes-Crisp-Stroud Approach to Spontaneous Emission", presented at this Conference, p. 635.

38. M.C. Newstein, Phys. Rev. *167*, 89 (1968); C.R. Stroud, Phys. Rev. A *3*, 1044 (1971).

39. E.T. Jaynes, *Statistical Physics*, Vol. 3, ed. K.W. Ford (W.A. Benjamin Inc., New York, 1963) Ch. 4; see particularly Eq.(21).

40. M. von Laue, Ann. d. Phys. *47*, 853; *48*, 668 (1915).