

Bayesian Analysis Users Guide  
Release 4.00, Manual Version 1

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## Appendix G

# 4dfp Overview

Files stored in the images subdirectory ending in 4dfp.img are 4dfp (4-dimensional floating point) format images. The corresponding file ending in 4dfp.ifh are the image file headers. The 4dfp.img files are purely binary files containing 4 dimensional arrayed images. The images stored in the 4dfp.img file are a single UNIX binary file containing a stack of images. The four dimensions are the array, slice, readout and phase encode. Symbolically, the loops needed to correctly read and store an image on a Big Endian machine are given by:

```
Do CurEle = 1, ArrayDim           ! loop over the array dimension
  Do CurSlice = 1, NoSlices       ! loop over the slice dimension
    Do CurRo = 1, NoRo           ! loop over the readout dimension
      Do CurPe = 1, NoPe         ! loop over the phase encodes

        Read the next 4 byte Big Endian number and place it in "Work"

        Copy "Work" into the image matrix

        Images(CurEle, CurPe, CurRo, CurSlice) = Work

      EndDo
    EndDo
  EndDo
EndDo
```

where ArrayDim is the size of the array, NoSlices is the number of slices, NoRo is the size of the image in the readouts direction and NoPe is the size of the image in the phase encode dimension. The images used in the Bayesian Analysis software are stored in Big Endian format and it does not matter what hardware platform writes the images: images are always written in Big Endian regardless of the hardware. If your reading one of our output images on a Little Endian machine, for example a PC, you must swap the byte order. Failure to do so will give essentially meaningless numbers in your image.

The 4dfp.img file contains the binary image, however that binary cannot be read without first parsing the associated 4dfp.ifh file. The 4dfp.ifh file is a separate image file header and it contains

among other things the dimensions of the images, Fig. G.1 is an example of this file. Most of the items in this header are pretty straight-forward, however we are going to dwell on the matrix size and the scaling factors a bit because unless you understand these you will not be able to correctly display an image. First, the four elements in the matrix size are: [1] the number of pixels in the phase encode (x domain), [2] the number of pixels in the readout (y domain), [3] the number of slices and [4] the size of the array variable. Similarly, the three scaling factors are the image pixels sizes in mm. the three scaling factors are the phase encode, the read out and the slice scaling factors. For our images the byte order will always be “bigendian”. Finally, the two file names are usually fully qualified path names to the appropriate files. Here they have been truncated to get them to paginate correctly.

Figure G.1: Example FDF File Header

```

INTERFILE                :=
version of keys          := 3.3
conversion program       := BayesPhase
program version          := 1.0
name of data file        := Bayes/test/images/LoadedImage_Abs.4dfp.ifh
source data file name    := Bayes.test.data/BayesPhase/image_IR.fid
patient ID               := N/A
date                     := 12-Oct-2011 13:50:29
number format            := float
number of bytes per pixel := 4
orientation              := 2
number of dimensions     := 4
matrix size [1]          := 128
matrix size [2]          := 128
matrix size [3]          := 1
matrix size [4]          := 5
scaling factor (mm/pixel) [1] := 0.15625
scaling factor (mm/pixel) [2] := 0.15625
scaling factor (mm/pixel) [3] := 0.15000000596046448
slice thickness (mm/pixel) := 0.15000000596046448
imagedata byte order     := bigendian
x label                   := Phase Encode (cm)
y label                   := Readout (cm)

```

Figure G.1 This is a example image file header (ifh file type) written by the linear phasing package. The image file header contains the parameters necessary to read a 4dfp.img file. The most critical part of the header are the matrix sizes and the scale factors. The matrix sizes are the phase encode, readout, number of slices and array size respectively. The three scaling factors are the pixel sizes in the images. In the Bayesian Analysis interface the real 4 byte numbers written in the image are always written in Big Endian form. The x and y labels are just text strings used to set the labels on the displayed images. Finally, the name of data file and source data file name are normally fully qualified path names to the appropriate files. Here, these names were truncated to get them to paginate correctly.



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