

Bayesian Analysis Users Guide  
Release 4.00, Manual Version 1

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## Appendix A

# Ascii Data File Formats

Ascii data files are used through out the entire Bayesian Analysis software. Often they are used for simple things like input to various packages. Sometimes they are used to loading Abscissa into plotting routines. And sometimes they are used to generate Fid and Image files. In all cases the exact file formats vary depending on the type of data and the package that is loading the Ascii file. This Appendix is a description of these file formats and how they are used. Most of the time the file formats are pretty simple and rather self explanatory and we will list a few of these shortly. However, in a few cases the actual file format of the Ascii data can be complicated. We give these more complicated file formats in this Appendix. For now here is a list of some of the more simple Ascii file formats:

**Images** can be loaded in Ascii. The interface Files/Load Image/Single-Column text file widget expects the data to be single column Ascii with each row in the image stacked one line after another.

**FID** data can be loaded as Ascii data. In this case one complex (real and imaginary) number are expected on each like of the FID. This type of data is used in the Files/Load Spectroscopic Fid/Text File widget.

**Ascii Input Files** for most packages, like an exponential package, allow Ascii data to be loaded. In these simple cases, the files are simply two column Ascii: one abscissa and one data column. However, see below because there are major exceptions to this rule.

### A.1 Ascii Input Data Files

The format of a general input Ascii file used in the Bayesian Analysis software is shown in Fig. A.1. Each line in an input Ascii file consist of three parts. This is illustrated by the double vertical lines in the figure. The first part is a single column plot abscissa, and as its name implies, this is the abscissa used in plotting the Ascii data. All plots of the Ascii data done by the interface are of the data verses the plot abscissa. If there are multiple data columns, then multiple plots are drawn by the interface. However, all of these data plots use the input plot abscissa on the current data set.

The second part of an Ascii input file, is the data and as indicated in Fig. A.1, there can be up to  $N$  data columns, where  $N$  is specified by the model, i.e., it is user defined. The number of these

Figure A.1: Ascii Data File Format

Plot Abscissa	Data Col 1	...	Data Col $N$	Abcissa Col 1	...	Abcissa Col $M$
1	25.1	...	30.2	1	...	3
2	15.1	...	10.2	2	...	-1
$\vdots$	$\vdots$	...	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Figure A.1: This table shows a schematic of a general input Ascii data file. The first column is simply a plot variable and in the case of a single column abscissa, this variable should be the abscissa. The columns labeled “Data Col 1” though “Data Col  $N$ ” are the  $N$  input data columns. The number of these column is implicit in the package model or it is defined explicitly in the model parameter file. Finally, when multiple abscissa’s are present, these abscissa follow the data columns. Here these  $M$  columns are labeled “Abcissa Col 1” through “Abcissa Col  $M$ ”. If there is only a single abscissa, then these abscissa columns are not present.

columns is often implicit in the nature of the package. For example, an inversion recovery data set would require only a single data column. However, a magnetization transfer packages might require real and imaginary data so the data section might have two columns. User defined models can be written that use any number of input data columns. In Fig. A.1 these data columns are labeled as “Data Col 1” through “Data Col  $N$ ” where there are  $N$  data columns.

Finally the computational abscissa section is present only when the package expects multicolumn abscissa. For example, in a diffusion tensor analysis each abscissa consist of a 3 dimensional  $B$  vector, so 3 abscissa are needed. When a model runs that uses a computational abscissa, the abscissa passed to the model program contains as many columns as specified by the model. For example, a diffusion tensor analysis using a  $B$  vector would be a 3 by  $n$  abscissa, where 3 is the number of abscissa columns and  $n$  is the number of data values in the current data set.

In Ascii packages multiple data sets can be loaded and analyzed jointly. Each data set can have a differing number of data values and different abscissa values. However, the number of abscissa columns is fixed for a given analysis. When the posterior probability is computed the model program must generate the model given the current parameters and abscissa. After this information is generated, the posterior probability for the current data set is computed and used in the Bayesian calculations. So while you could have different abscissa values in the different data sets, all data sets must use the same number of abscissa columns.

## A.2 Ascii Image File Formats

The previous section describes the file format for Ascii data files that are used as input to the various packages. However, Ascii images can also be loaded and in this case there are three distinct image formats, one single column, one multicolumn and one  $k$ -space Ascii format.

In the case of single column Ascii images, the images are stacked in the Ascii file one pixel at a time. If the image consists of  $N$  rows by  $M$  columns then the interface expects rows 1 (all  $M$  values) followed by row 2, etc until all  $N$  rows are read. Images can be stacked by slice or element number and you can specify in the popup how they are stacked so the outer loops can be either slice number or array element number.

Multicolumn Ascii files can also be loaded. In this case all elements in a give row are read from record one of the input file. Record 2 corresponds to the second row in the image, etc. The ordering of the images by slice and element is again under user control and can be specified when the images are loaded.

Finally, images can be complex  $k$ -spaced data. In this case, the complex  $k$ -space data is two column Ascii data. Each row of the complex data corresponds to one readout and readouts are stacked one after another. This complex  $k$ -space data is converted into a Varian fid. This fid is then Fourier transformed, phased, and displayed as a complex image. If multiple  $k$ -space images are present, the slice and element orders are again under user control. For  $k$ -space fid data the output from the interface is the real, imaginary and absolute value images.

All images are whether or not they are input as Ascii files,  $k$ -space files, or any other formats are copied into your current working directory. These images are located in BayesHome/WorkDir/images, where “BayesHome” is your current Bayes home directory, “WorkDir” is your current working directory, and “images” is the images subdirectory. Loaded images preserve the name of the name of the input image and duplicates will overwrite the image directory. However, loaded  $k$ -space data are first converted into a Varian fid. This fid is located in the BayesHome/WorkDir/Image.fid subdirectory. After this conversion, the  $k$ -space data is then Fourier transformed and displayed as an image. These images are output with the name “LoadedImage\_Real.4dfp.ima,” “LoadedImage\_Imag.fdfp.ima,” and “LoadedImage\_Abs.4dfp.ima” for the real, imaginary and absolute value images respectively.

In all of these cases, an abscissa file will almost certainly have to be loaded and in the case of  $k$ -space data is required for the conversion. The next section briefly discusses the format of an abscissa file used with images.

### A.3 The Abscissa File Format

If an image is arrayed, the values of the array variable must be specified in an abscissa file before any image processing can be done on the image. Additionally, when  $k$ -space images are loaded, the abscissa must be available to create the Varian fid file. These abscissa values are given in multicolumn Ascii files, one column for each abscissa column. Unusually abscissa are one dimensional like, for example, in an inversion recovery experiment. In which case the abscissa file is a simple single column Ascii file. But sometimes they can be much more complicated, for example, a  $B$  matrix diffusion experiment would have to have a 6 column abscissa, one entry for each of the 6 independent  $B$  values in a diffusion experiment. In all cases the actual meaning of the columns in an abscissa are specified by the model used in processing the image data. For example a diffusion tensor image would have an abscissa consisting of the gradient or  $B$  values used in the experiment. These gradient or  $B$  values could be one, two, etc. up to 6 dimensional depending on the problem. So the abscissa file could be one column, as in diffusion along a single direction, two column, for diffusion in a plane, all the way up to a 6 column abscissa for a full  $B$  matrix diffusion tensor experiment.



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