Bayesian Analysis Users Guide
Release 4.00, Manual Version 1

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Chapter 28

Analyze Image Pixel

The Analyze Image Pixel package allows you to enter a model of your own and then use Bayesian probability theory to analyze that model.[1] The Java interface to the Image Pixel package is shown in Fig. 28.1. The Bayesian calculations performed by this package are identical to those in Bayes Enter Ascii, Chapter 20, and consequently we are not going to repeat those calculation here; rather in this Chapter we will concentrate our attention on the problem of how to use this package. To use this package, you must do the following:

Select the "Analyze Image Pixel" package from the Package menu.

Load the image data that is to be processed by the package. The analyze image Pixel package analyzes arrayed images on a pixel by pixel basis.

Load a Fortran or C model using the "System" or "User" buttons in the "Load And Build Model" widget group.

Load an abscissa file. A typical arrayed image data set is a stack of images each gathered at some parameter settings. For example, if the data are diffusion tensor data, then the abscissa would be a vector specifying the b values for each element in the array. These b values would be b_x, b_y and b_z. So the abscissa file would be a three column Ascii file containing the b values each element in the array. In general the user specifies the number of abscissa columns, See Chapter A.3 for more on the abscissa file. Note the Analyze Image Pixel package uses the same abscissa for every pixel in the data.

Build the model using the "Build" button.

Check the Analysis Options boxes as you see fit.

Find Outliers tells the package to use an outlier model to select and eliminate pixels with odd characteristics.

[1] I would like to build a system library of predefined models. If you have models that you think would be of general use, I would like to hear from you. To have one of your models included, I would need the source code, the parameter file, a brief description of the model equations and data requirements

Figure 28.1: The Interface To The Analyze Image Pixels Package

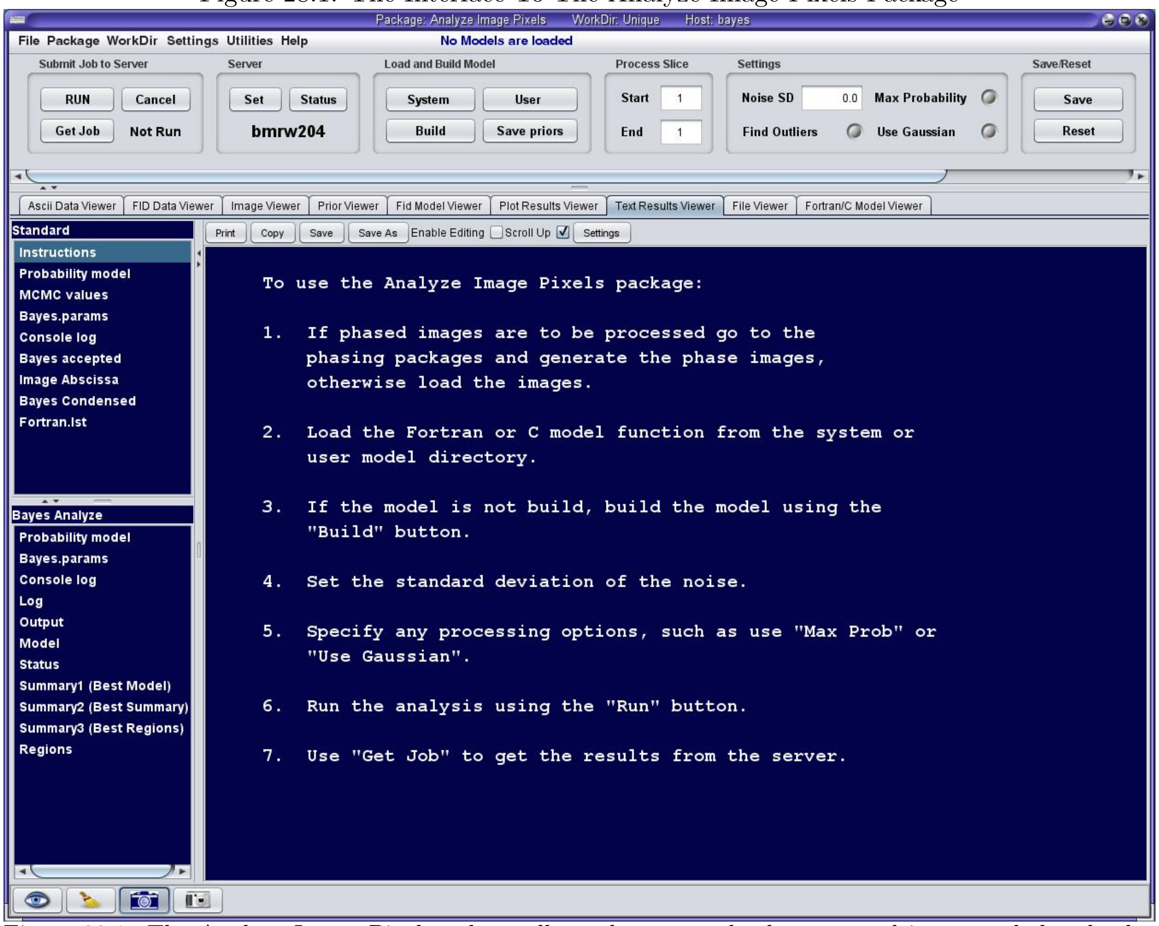

Figure 28.1: The Analyze Image Pixel package allows the user to load an arrayed image and then load a model, either system defined or user defined, and analyze the input image on a pixel by pixel basis using the loaded model. Prior to running an analysis, you must also load an appropriate Abscissa file. Unlike Ascii input data, which requires an Abscissa to load, any image can be loaded. So to process the data the Abscissa values must be loaded.

Max Probability tells the package to locate the maximum of the posterior probability. Locating the maximum is done using a searching algorithm and is much faster than using a Markov chain Monte Carlo algorithm. When Max Probability is selected the output images are generated using the parameters that maximized the posterior probability.

Use Gaussian tells the package to switch from using a t-distribution to using a Gaussian. In this case the input standard deviation of the noise is used as the standard deviation of the Gaussian likelihood. If Use Gaussian is not selected, a Students' t-distribution is used for the likelihood. The input noise standard deviation is used to threshold pixels. Arrays of pixels having a root mean-square smaller then the input standard deviation of the noise are not processed.

Set the Noise SD to a value equal to your best estimate of the standard deviation of the noise in the image stack that is to be processed. To get an estimate of the noise standard deviation, draw an ROI in a region where there is no signal and hit the "Get Statistics" button. The output on the right-hand side labeled "RMS" contains the root mean-square image pixel value and is a good estimate of the noise standard deviation.

Review the prior probabilities for the loaded model using the Prior Viewer.

Select the server that is to process the analysis.

Check the status of the selected server to determine if the server is busy, change to another server.

Run the the analysis on the selected server by activating the Run button.

Get the the results of the analysis by activating the Get Job button. If the analysis is running, this button will return the Accepted report containing the status of the current run. Otherwise, it will fetch and display the results from the current analysis.

The actual processing done by this package are essentially the same as that done in the Bayes Enter Ascii package and we refer you to that package, Chapter 20, for more on the processing being done. However, there is one difference: the Analyze Image Pixel package does a model selection calculation. In this calculation there are two models: a No Signal model, and a signal model. The No Signal model assumes the signal consists of only noise while the signal model assumes the signal is given by your model plus noise. When the no signal model is selected, there are no output parameters. Output parameter maps contain zero in these no signal regions. However, there is an exception to this, on the output map containing the standard deviation of the noise, both signal and no signal regions have an estimate of the noise standard deviation. In regions with no signal, the standard deviation is computed as the root mean-square of the total data value. In regions containing signals, the output standard deviation is computed as the root means-square residual.

28.1 Modification History

In release 4.10, the Markov chain Monte Carlo routine in Bayes Image Pixels was replaced by one as similar to the one in the Enter Ascii Model package, Chapter 20, as possible. This was done because in some cases the Enter Ascii package's Markov chain was able to solve problems that Bayes Image Pixel package could not. Given that the data and the model were identical in the cases at hand, I concluded the Markov chain in Bayes Image Pixels package needed replacing. The new routine is almost line by line identical to the routine in the Enter Ascii package.

In addition to replacing the Markov chain, I also modified the output routines so that it tests each output image to see if it is null, i.e., all zero. If the image is zero everywhere nothing is written. This was done to eliminate a perceived problem. In the process of testing a new model, a user generated an image containing no signal and then ran the Bayes Image Pixel using the no signal image and his new model. The old Bayes Image Pixel package correctly identified the pixels as no signal and so set the output images to zero. When processing was complete, the output routines wrote these zero images, and the users, myself included, interpreted these zero output images as an error on the part of Bayes Image Pixels package. That interpretation was incorrect and as soon as I realized what was happening, I modified the output routines so that zero images are not written.

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