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Resonant Modes in Waveguide Windows*

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Summary—Analysis and experimental verification of a class of resonant fields, called ghost-modes, occurring in waveguide dielectric windows are presented. Numerical solutions for a simple geometry are given through universal curves. Knowledge about ghost-modes has importance to designers of high-power windows. It also leads to a measuring technique for dielectric constants through a frequency measurement.

Introduction

THE general phenomenon of ghost-modes in imperfect waveguides, special cases of which have been noted before, was predicted by one of the authors.[1] The present paper presents a quantitative analysis and confirming experiments of a class of ghost-mode resonances occurring in a particularly simple waveguide window, where exact analysis, using transmission-line theory, is applicable. A ghost-mode is a resonant electromagnetic field configuration, existing in the vicinity of certain waveguide obstacles, such as dielectric windows. Its transverse field configuration is that of an ordinary waveguide mode and its resonant frequency lies below the cutoff frequency of the particular mode in the unperturbed guide. Thus, the ghost-mode fields decay exponentially on either side of the waveguide obstacle and no energy travels away. Within the region of the obstacle the z-variation of the fields must have oscillatory character.

Analysis

A window configuration simple enough to allow exact analysis is shown in Fig. 1. The dielectric slab shall be homogeneous and isotropic; the surrounding waveguide shall be straight and lossless, but its cross-sectional shape may be arbitrary. Under these assumptions the window does not introduce modal conversions, and analysis may proceed using conventional transmission-line theory.

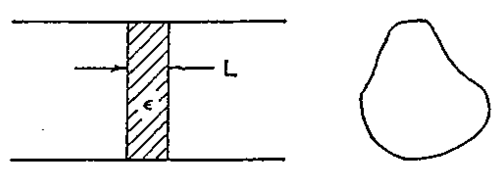

Fig. 1—Transverse dielectric slab window.

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[1] E. T. Jaynes, "Ghost modes in imperfect waveguides," Proc. IRE, vol. 46, pp. 415–418; February, 1958. (Note that Fig. 2 in this reference was incorrectly drawn; the curves should be rotated 180° in the plane of the paper, about an axis passing through the center of the diagram.)

wave impedance

$$Z_{wn} = Z_0 \begin{Bmatrix} k/k_{3n} \\ k_{3n}/k \end{Bmatrix}, \tag{1}$$

where the upper and lower expressions within curly brackets relate to TE and TM modes, respectively. The quantities Z_0 and k are the free-space impedance and the free-space propagation constant. The waveguide propagation constant of the nth mode is $k_{3n} = \sqrt{k^2 - k_{cn}^2}$. We shall be interested in frequencies below waveguide cutoff. Here it is convenient to replace jk_{3n} by the real, positive quantity $k_{3n}' = \sqrt{k_{cn}^2 - k^2}$ in all expressions. The wave impedance, for example, becomes

$$Z_{wn} = jZ_0 \begin{Bmatrix} k/k_{3n}' \\ -k_{3n}'/k \end{Bmatrix}. \tag{2}$$

In the guide completely filled with material of dielectric constant ϵ, one has

$$Z_{wn}^{(\epsilon)} = Z_0 \begin{Bmatrix} k/\beta_n \\ \beta_n/\epsilon k \end{Bmatrix}, \tag{3}$$

where $\beta_n = \sqrt{\epsilon k^2 - k_{cn}^2}$ is the propagation constant of the nth mode in the dielectric filled guide.

The geometrical configuration (Fig. 1) has a symmetry plane $z=0$. Any resonant fields, therefore, have symmetry properties with respect to $z=0$, which may be of an even or odd character. Analytically, advantage is taken of this fact by considering only the region $z>0$ under the condition of an electric or magnetic short circuit at $z=0$.

Electric Short at $z=0$: $Z_{wn}(0)=0$

Under this condition, the impedance at the dielectric-air interface ($z=L/2$) is

$$Z_{wn}(L/2) = -jZ_{wn}^{(\epsilon)} \tan \beta_n L/2. \tag{4}$$

Continuity of tangential field components at the dielectric-air interface requires continuity of wave impedance,

$$Z_{wn}(L/2) = Z_{wn}. \tag{5}$$

Substituting (2)–(4) into (5) yields

$$\tan \beta_n L/2 = \begin{Bmatrix} -\beta_n/k_{3n}' \\ \epsilon k_{3n}'/\beta_n \end{Bmatrix}. \quad \begin{matrix} (6a) \\ (6b) \end{matrix}$$

Frequencies that satisfy this equation are the ghost-mode resonant frequencies.

The impedance at the dielectric-air interface becomes

$$Z_{wn}(L/2) = jZ_{wn}^{(\epsilon)} \cot \beta_n L/2, \tag{7}$$

and the continuity condition of tangential fields at $z=L/2$ leads to

$$\cot \beta_n L/2 = \begin{Bmatrix} \beta_n/k_{3n}' \\ -\epsilon k_{3n}'/\beta_n \end{Bmatrix}. \quad \begin{matrix} (8a) \\ (8b) \end{matrix}$$

Eqs. (6b) and (8a) always have at least one real solution. A field analysis shows that these solutions are characterized by *even* symmetry of their longitudinal field component. They will be called even modes, and the number N_e of such solutions is determined by

$$N_e - 1 < \sqrt{\epsilon - 1}\,\frac{k_{cn}L}{2\pi} < N_e. \tag{9}$$

On the other hand, (6a) and (8b) have real solutions only when ϵ and L exceed certain minimum values. Solutions obtained in such cases exhibit *odd* symmetry of their longitudinal field component, and will be called odd modes. In general, there exist N_0 odd modes, if

$$N_0 < \sqrt{\epsilon - 1}\,\frac{k_{cn}L}{\pi} < N_0 + 1. \tag{10}$$

Resonant field patterns corresponding to the lowest frequency solutions for some simple waveguide modes are sketched in Fig. 2. The upper-right and lower-left hand patterns correspond to (6), the others correspond to (8).

Eqs. (6) and (8) may be solved graphically by our plotting both sides of the equations as functions of frequency, k/k_{cn}, and determining the points of intersection. Such solutions, obtained for various values of ϵ and $k_{cn}L$, are shown in Figs. 3 and 4. Typically, the TE resonant frequencies decrease faster than the TM resonant frequencies, as k_cL is increased. All curves approach the value $(k/k_{cn})_{res} = \epsilon^{-1/2}$ for very large $k_{cn}L$. This is true for both TE and TM resonant modes and is plausible from the fact that for very large $k_{cn}L$ practically the entire field is confined to the inside of the dielectric, and resonances occur, in a well-known manner, when the dielectric is an integral number of half wavelengths long.

Experimental Verification of Ghost-Modes

The ghost-mode resonances derived above are experimentally verified by an arrangement shown in Fig. 5. A window sample was placed into the center of a three-inch ID circular waveguide section. Both ends of the approximately two-foot long section were left open, since the modes of interest are below waveguide cutoff and have substantially decayed when reaching the open

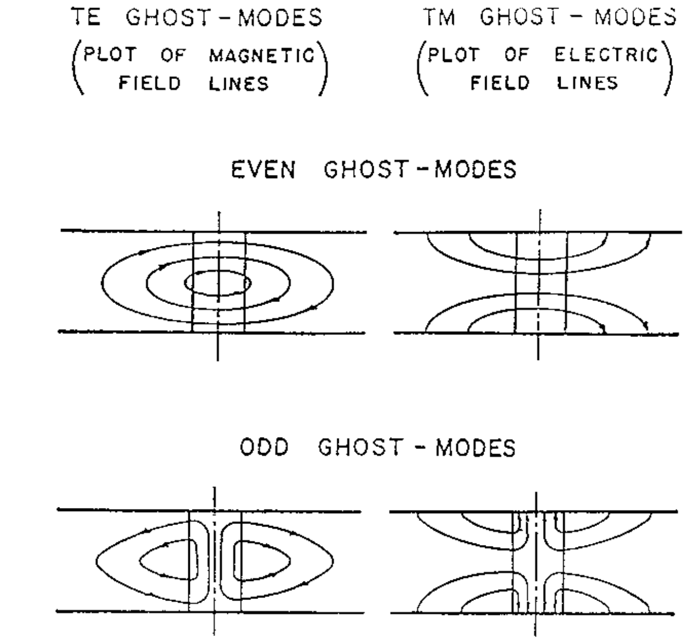

Fig. 2—Sketch of the lowest TE and TM ghost-mode field patterns of an even and an odd symmetry. (The transverse field variations shown apply to TE_{10} and TM_{11} in rectangular guide or TE_{11} and TM_{01} in circular guide.)

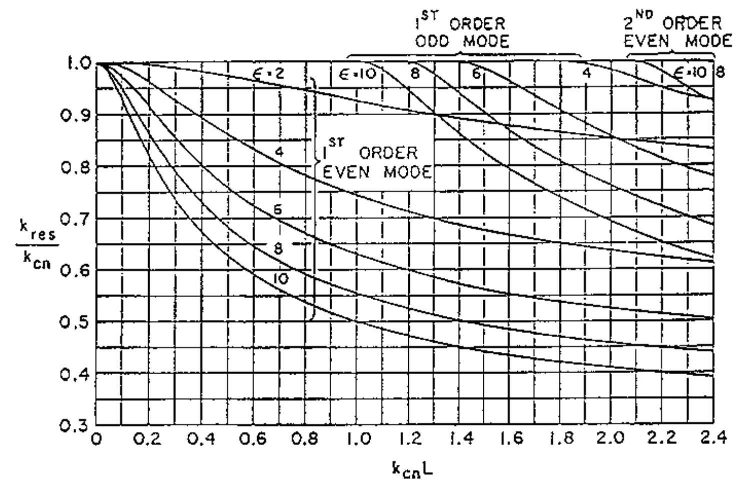

Fig. 3—Universal curves showing TE ghost-mode resonant frequencies of a transverse dielectric slab.

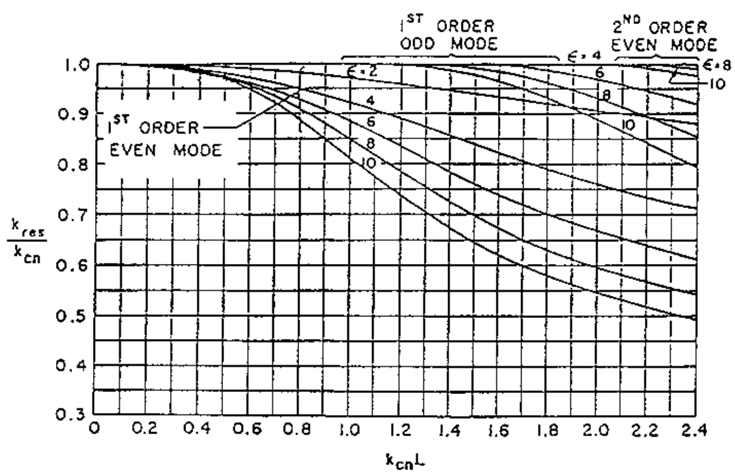

Fig. 4—Universal curves showing TM ghost-mode resonant frequencies of a transverse dielectric slab.

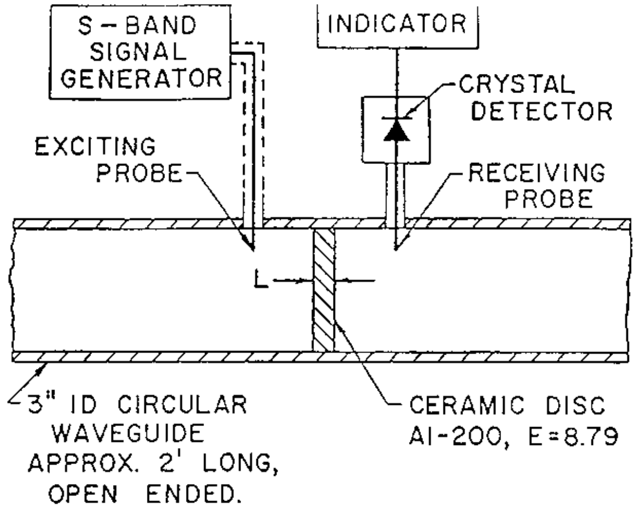

Fig. 5—Ghost-mode experiment.

ends. Microwave energy was loosely coupled to the guide through an electric probe at the guide wall, approximately four inches away from the dielectric. A similar probe, located on the opposite side of the dielectric, was connected to a crystal rectifier and an indicator. The coupling between the probes is small, so that strong indicator deflections occur only at resonance conditions in the guide. Such resonances, observed below 3 kmc, were identified as TE_{11} and TM_{01} ghost-modes, by observing the transverse symmetries of the residual fields at the open guide ends with a small perturbation rod. To vary the slab thickness L, one or more ceramic discs ($\epsilon = 8.79$) of $\frac{1}{8}$-inch thickness were stacked up. The observed ghost-mode frequencies are marked on Fig. 6, which also contains the theoretical curves for the particular arrangement as a comparison.

The approximate Q factors indicated were determined from the half-power bandwidth of the resonances. The Q factors generally decrease for larger slab thickness because the resonant fields become more and more confined to the volume of the relatively lossy dielectric.

The good agreement between theoretical and experimental data in Fig. 6 may be taken as a confirmation of the fact that the dielectric constant is known to good accuracy. Conversely, one might well make use of the described experiment to determine unknown dielectric constants of slabs by measuring a ghost-mode resonant frequency. Error analysis applied to the lowest TE_{11} ghost-mode in our experiment yields

$$\frac{\Delta\epsilon}{\epsilon} = -15.5\left(\frac{\Delta f}{f}\right)_{res}$$

for a slab of $\frac{1}{8}$-inch thickness (neglecting geometrical errors). Considering the precision attainable in frequency measurements, this appears encouraging. If the dielectric constant of the slab is large, a possible air gap δ between the dielectric and the waveguide wall may be-

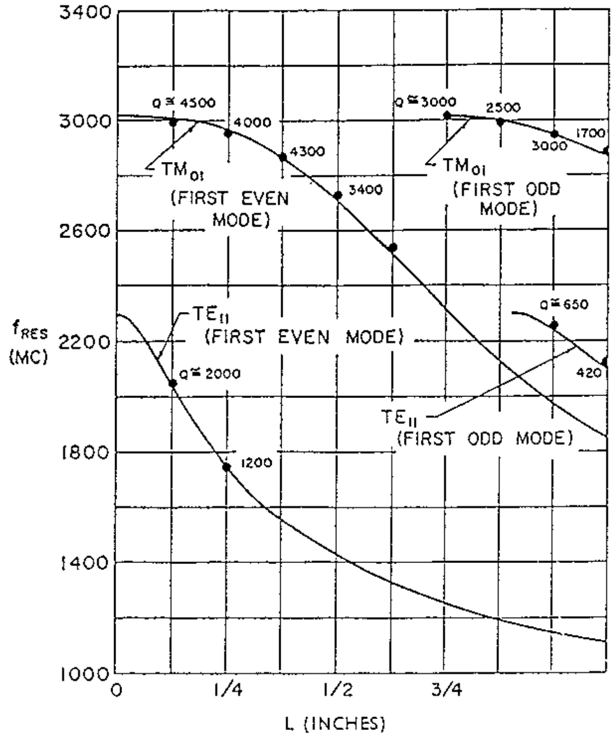

Fig. 6—Ghost-mode resonant frequencies of ceramic discs ($\epsilon = 8.79$) of various thickness L. Curves are computed, dots are measured.

come troublesome. It may introduce an error of the following order of magnitude:

$$\left(\frac{\Delta f}{f}\right)_{\text{res}} = \frac{1}{2} \frac{\epsilon(\delta/a)}{1 + \epsilon(\delta/a)},$$

where a is a transverse waveguide dimension, *e.g.*, waveguide radius.

Conclusion

The phenomenon of resonant ghost-modes existing in the vicinity of a dielectric window has been discussed both theoretically and experimentally for the simplest geometry available. In practice, the ghost-mode resonances of some higher-order waveguide modes may coincide with the operating frequency of the dominant waveguide mode. Since these modes are orthogonal, no coupling would be expected under ideal conditions. However, imperfections, such as a slight tilt of the window, uneven thickness, or inhomogeneous dielectric may provide the modal coupling.

Waveguide discontinuities (irises, couplers, bends, etc.) located in the vicinity of the window may also provide this function. Since the ghost-mode resonances have high Q, only a small amount of coupling is required to produce appreciable resonant fields.

High-power microwave signals are very likely accompanied by harmonics. These harmonic frequencies may coincide with ghost-mode resonances, and coupling may be provided by imperfections or discontinuities.

Appreciable excitation of ghost-modes may considerably lower the breakdown power of a waveguide window. Rather than minimizing the coupling to these modes by close tolerances, the window designer may find the information of Figs. 3 and 4 useful to avoid the existence of ghost-modes within the operating frequency range.

Ghost-mode resonances may exist in dielectric windows of more complicated geometry than that discussed above. Examples include slanted dielectric plates and ceramic cones. Analysis of such configurations becomes complicated. Transmission-line theory is no longer useful, but the mathematical tool of a normal-mode expansion may be employed and yields approximate results.[2] Such windows of increased complexity generally couple many waveguide modes together. A ghost-mode is, therefore, no longer a pure waveguide mode in its transverse field configuration. Moreover, if such a ghost-mode is strongly coupled to the propagating dominant waveguide mode, its external Q factor may become rather small, so that the resonance is less pronounced.

The existence of ghost-modes in dielectric obstacles of arbitrary shape may be made plausible by the following: the insertion of dielectric material ($\epsilon > 1$) into the guide effects an increase in "electrical cross section" of the guide, so that a wave may have propagating character (*i.e.*, oscillatory z-variation) within the length L of the obstacle, while it decays outside. The frequencies at which the impedance boundary conditions on both sides of the obstacle can be met are the ghost-mode resonances.

Analysis of ghost-modes for any given window geometry makes it possible to determine the dielectric constant of the material through a frequency measurement. Even without analysis, a ghost-mode resonance measurement may be useful as a uniformity test of dielectric samples.

It is of interest to note the behavior of two windows (of the kind shown in Fig. 1) placed in the same waveguide. If the distance between them is such that the ghost-mode fields overlap, there appear two resonant frequencies, one being slightly higher, the other slightly lower than the original ghost-mode resonance. The situation is analogous to coupled resonant tanks. It may be analyzed by use of the same methods as were employed analyzed using the same methods as were employed above. Such analysis and corresponding experiments have been performed and show good agreement. This experiment is particularly interesting as it represents one of the rare cases where the coupling between resonant circuits may be found easily and accurately through analysis.

Finally, it might be noted that the location of ghost-mode resonances represents a simple but unusually interesting laboratory experiment by which several aspects of waveguide theory can be demonstrated in the teaching of microwave techniques.

[2] M. P. Forrer, "On the Boundary Value Problem of Waveguide Windows," W. W. Hansen Labs. of Physics, Stanford University, Stanford, Calif., Microwave Lab. Rept. No. 575; March, 1959.