PROBABILITY THEORY

with Applications in Science and Engineering

A Series of Informal Lectures

by

E. T. Jaynes

Professor of Physics

Washington University

St. Louis, Missouri 63130

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*The following notes represent what is completed to date of a projected book manuscript. All Lectures after No. 9 are incomplete; Lectures 11 and 12 are missing entirely, although their content is already largely published in E. T. Jaynes, "Prior Probabilities," IEEE Trans. Syst. Sci. and Cybern. SSC-4, Sept. 1968, pp. 227-241; and "The Well-Posed Problem," in Foundations of Physics, 3, 477 (1973). The projected work will contain approximately 30 Lectures; in the meantime, comments are solicited on the present material.

SUMMARY OF BASIC RULES AND NOTATION

Deductive Logic (Boolean Algebra): Denote propositions by A, B, etc., their denials by a ≡ "A is false," etc. Define the logical product and logical sum by

$$AB \equiv \text{"Both A and B are true."}$$

$$A+B \equiv \text{"At least one of the propositions, A, B is true."}$$

Deductive reasoning then consists of applying relations such as $AA = A$; $A(B+C) = AB + AC$; $AB+a = ab+B$; if $D = ab$, then $d = A+B$, etc., in which the = sign denotes equal "truth value."

Inductive Logic (probability theory): This is an extension of deductive logic, describing the reasoning of an idealized being (our "robot"), who represents degrees of plausibility by real numbers:

$$(A|B) = \text{probability of A, given B.}$$

Elementary requirements of common sense and consistency, such as: (a) if a conclusion can be reasoned out in more than one way, every possible way must lead to the same result; and (b) in two problems where the robot has the same state of knowledge, he must assign the same probabilities, then uniquely determine these basic rules of reasoning (Lect. 3):

Rule 1: $(AB|C) = (A|BC)(B|C) = (B|AC)(A|C)$

Rule 2: $(A|B) + (a|B) = 1$

Rule 3: $(A+B|C) = (A|C) + (B|C) - (AB|C)$

Rule 4: If $\{A_1 \ldots A_n\}$ are mutually exclusive and exhaustive, and information B is indifferent to them; i.e., if B gives no preference to one over any other, then

$$(A_i|B) = 1/n, \quad i = 1, 2, \ldots, n$$

Corollaries: From Rule 1 we obtain Bayes' theorem:

$$(A|BC) = (A|C)\frac{(B|AC)}{(B|C)}$$

From Rule 3, if $\{A_1 \ldots A_n\}$ are mutually exclusive,

$$(A_1 + \ldots + A_n|B) = \sum_{i=1}^{n}(A_i|B)$$

If in addition the A_i are exhaustive, we obtain the chain rule:

$$(B|C) = \sum_{i=1}^{n}(BA_i|C) = \sum_{i=1}^{n}(B|A_iC)(A_i|C)$$

These are the relations most often used in practical calculations.

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