PROBABILITY THEORY

with Applications in Science and Engineering

A Series of Informal Lectures

by

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*The following notes represent what is completed to date of a projected book manuscript. All Lectures after No. 9 are incomplete; Lectures 11 and 12 are missing entirely, although their content is already largely published in E. T. Jaynes, "Prior Probabilities," IEEE Trans. Syst. Sci. and Cybern. SSC-4, Sept. 1968, pp. 227-241; and "The Well-Posed Problem," in Foundations of Physics, 3, 477 (1973). The projected work will contain approximately 30 Lectures; in the meantime, comments are solicited on the present material.
SUMMARY OF BASIC RULES AND NOTATION

**Deductive Logic (Boolean Algebra):** Denote propositions by A, B, etc., their denials by a = "A is false," etc. Define the logical product and logical sum by

\[ AB \equiv \text{"Both } A \text{ and } B \text{ are true."} \]
\[ A+B \equiv \text{"At least one of the propositions, } A, B \text{ is true."} \]

Deductive reasoning then consists of applying relations such as AM = A; A(B+C) = AB + AC; AB+a = ab+B; if D = ab, then d = A+B, etc., in which the = sign denotes equal "truth value."

**Inductive Logic (probability theory):** This is an extension of deductive logic, describing the reasoning of an idealized being (our "robot"), who represents degrees of plausibility by real numbers:

\[ (A|B) = \text{probability of } A, \text{ given } B. \]

Elementary requirements of common sense and consistency, such as: (a) if a conclusion can be reasoned out in more than one way, every possible way must lead to the same result; and (b) in two problems where the robot has the same state of knowledge, he must assign the same probabilities, then uniquely determine these basic rules of reasoning (Lect. 3):

**Rule 1:** \[ (AB|C) = (A|BC)(B|C) = (B|AC)(A|C) \]
**Rule 2:** \[ (A|B) + (a|B) = 1 \]
**Rule 3:** \[ (A+B|C) = (A|C) + (B|C) - (AB|C) \]
**Rule 4:** If \( \{A_1,...,A_n\} \) are mutually exclusive and exhaustive, and information \( B \) is indifferent to them; i.e., if \( B \) gives no preference to one over any other, then

\[ (A_i|B) = 1/n, \text{ } i = 1, 2, ..., n \]

**Corollaries:** From Rule 1 we obtain Bayes' theorem:

\[ (A|BC) = (A|C) \frac{(B|AC)}{(B|C)} \]

From Rule 3, if \( \{A_1,...,A_n\} \) are mutually exclusive,

\[ (A_1 + ... + A_n|B) = \sum_{i=1}^{n} (A_i|B) \]

If in addition the \( A_i \) are exhaustive, we obtain the chain rule:

\[ (B|C) = \sum_{i=1}^{n} (BA_i|C) = \sum_{i=1}^{n} (B|A_iC)(A_i|C) \]

These are the relations most often used in practical calculations.

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