

Bayesian Analysis Users Guide
Release 4.00, Manual Version 1

G. Larry Bretthorst
Biomedical MR Laboratory
Washington University School Of Medicine,
Campus Box 8227
Room 2313, East Bldg.,
4525 Scott Ave.
St. Louis MO 63110
<http://bayes.wustl.edu>
Email: larry@bayes.wustl.edu

August 21, 2013

Contents

Manual Status	14
1 An Overview Of The Bayesian Analysis Software	17
1.1 The Server Software	17
1.2 The Client Interface	20
1.2.1 The Global Pull Down Menus	22
1.2.2 The Package Interface	22
1.2.3 The Viewers	25
2 Installing the Software	27
3 the Client Interface	29
3.1 The Global Pull Down Menus	31
3.1.1 the Files menu	31
3.1.2 the Packages menu	36
3.1.3 the WorkDir menu	41
3.1.4 the Settings menu	42
3.1.5 the Utilities menu	46
3.1.6 the Help menu	47
3.2 The Submit Job To Server area	47
3.3 The Server area	48
3.4 Interface Viewers	49
3.4.1 the Ascii Data Viewer	49
3.4.2 the fid Data Viewer	51
3.4.3 Image Viewer	56
3.4.3.1 the Image List area	56
3.4.3.2 the Set Image area	58
3.4.3.3 the Image Viewing area	58
3.4.3.4 the Grayscale area on the bottom	60
3.4.3.5 the Pixel Info area	60
3.4.3.6 the Image Statistics area	60
3.4.4 Prior Viewer	62
3.4.5 fid Model Viewer	65
3.4.5.1 The fid Model Format	65

3.4.5.2	The Fid Model Reports	67
3.4.6	Plot Results Viewer	68
3.4.6.1	the Data, Model and Residuals Plots	70
3.4.6.2	the Posterior Probabilities Plots	71
3.4.7	the Posterior Probability Vs Parameter Samples plot	72
3.4.7.1	the Expected Log Likelihood Plot	75
3.4.7.2	the Scatter Plots	75
3.4.7.3	the Log Probability Plot	78
3.4.8	Text Results Viewer	80
3.4.9	Files Viewer	86
3.4.10	Fortran/C Code Viewer	86
3.4.10.1	Fortran/C Model Viewer Popup Editor	88
4	An Introduction to Bayesian Probability Theory	91
4.1	The Rules of Probability Theory	91
4.2	Assigning Probabilities	94
4.3	Example: Parameter Estimation	101
4.3.1	Define The Problem	102
4.3.1.1	The Discrete Fourier Transform	102
4.3.1.2	Aliases	105
4.3.2	State The Model—Single-Frequency Estimation	106
4.3.3	Apply Probability Theory	107
4.3.4	Assign The Probabilities	110
4.3.5	Evaluate The Sums and Integrals	112
4.3.6	How Probability Generalizes The Discrete Fourier Transform	115
4.3.7	Aliasing	118
4.3.8	Parameter Estimates	124
4.4	Summary and Conclusions	127
5	Given Exponential Model	129
5.1	The Bayesian Calculation	131
5.2	Outputs From The Given Exponential Package	133
6	Unknown Number of Exponentials	135
6.1	The Bayesian Calculations	137
6.2	Outputs From The Unknown Number of Exponentials Package	140
7	Inversion Recovery	143
7.1	The Bayesian Calculation	145
7.2	Outputs From The Inversion Recovery Package	146
8	Bayes Analyze	147
8.1	Bayes Model	151
8.2	The Bayes Analyze Model Equation	153
8.3	The Bayesian Calculations	159
8.4	Levenberg-Marquardt And Newton-Raphson	163

8.5	Outputs From The Bayes Analyze Package	167
8.5.1	The “bayes.params.nnnn” and “bayes.model.nnnn” Files	169
8.5.1.1	The Bayes Analyze File Header	169
8.5.1.2	The Global Parameters	174
8.5.1.3	The Model Components	175
8.5.2	The “bayes.output.nnnn” File	177
8.5.3	The “bayes.probabilities.nnnn” File	181
8.5.4	The “bayes.log.nnnn” File	184
8.5.5	The “bayes.status.nnnn” and “bayes.accepted.nnnn” Files	187
8.5.5.1	The “bayes.model.nnnn” File	188
8.5.6	The “bayes.summary1.nnnn” File	189
8.5.7	The “bayes.summary2.nnnn” File	190
8.5.8	The “bayes.summary3.nnnn” File	191
8.6	Bayes Analyze Error Messages	192
9	Big Peak/Little Peak	197
9.1	The Bayesian Calculation	199
9.2	Outputs From The Big Peak/Little Peak Package	206
10	Metabolic Analysis	209
10.1	The Metabolic Model	213
10.2	The Bayesian Calculation	215
10.3	The Metabolite Models	218
10.3.1	The IPGD-D2O Metabolite	218
10.3.2	The Glutamate.2.0 Metabolite	222
10.3.3	The Glutamate.3.0 Metabolite	225
10.4	The Example Metabolite	226
10.5	Outputs From The Bayes Metabolite Package	228
11	Find Resonances	229
11.1	The Bayesian Calculations	231
11.2	Outputs From The Bayes Find Resonances Package	236
12	Diffusion Tensor Analysis	237
12.1	The Bayesian Calculation	239
12.2	Using The Package	244
13	Big Magnetization Transfer	249
13.1	The Bayesian Calculation	249
13.2	Outputs From The Big Magnetization Transfer Package	252
14	Magnetization Transfer	255
14.1	The Bayesian Calculation	257
14.2	Using The Package	261

15 Magnetization Transfer Kinetics	267
15.1 The Bayesian Calculation	269
15.2 Using The Package	273
16 Given Polynomial Order	277
16.1 The Bayesian Calculation	279
16.1.1 Gram-Schmidt	279
16.1.2 The Bayesian Calculation	280
16.2 Outputs From the Given Polynomial Order Package	282
17 Unknown Polynomial Order	285
17.1 Bayesian Calculations	287
17.1.1 Assigning Priors	288
17.1.2 Assigning The Joint Posterior Probability	289
17.2 Outputs From the Unknown Polynomial Order Package	291
18 Errors In Variables	295
18.1 The Bayesian Calculation	297
18.2 Outputs From The Errors In Variables Package	300
19 Behrens-Fisher	303
19.1 Bayesian Calculation	303
19.1.1 The Four Model Selection Probabilities	306
19.1.1.1 The Means And Variances Are The Same	307
19.1.1.2 The Mean Are The Same And The Variances Differ	309
19.1.1.3 The Means Differ And The Variances Are The Same	310
19.1.1.4 The Means And Variances Differ	311
19.1.2 The Derived Probabilities	312
19.1.3 Parameter Estimation	313
19.2 Outputs From Behrens-Fisher Package	314
20 Enter Ascii Model	321
20.1 The Bayesian Calculation	323
20.1.1 The Bayesian Calculations Using Eq. (20.1)	323
20.1.2 The Bayesian Calculations Using Eq. (20.2)	324
20.2 Outputs Form The Enter Ascii Model Package	327
21 Test Your Own ASCII Model	329
22 Ascii Model Selection	331
23 Phasing An Image	333
23.1 The Bayesian Calculation	334
23.2 Using The Package	340

24 Phasing An Image Using Non-Linear Phases	343
24.1 The Model Equation	343
24.2 The Bayesian Calculations	345
24.3 The VnmrJ and Vnmr Interfaces	347
28 Analyze Image Pixel	361
28.1 Modification History	363
29 Image Pixel Model Selection	365
A Ascii Data File Formats	367
A.1 Ascii Input Data Files	367
A.2 Ascii Image File Formats	368
A.3 The Abcissa File Format	369
B Markov chain Monte Carlo With Simulated Annealing	375
B.1 Metropolis-Hastings Algorithm	376
B.2 Multiple Simulations	377
B.3 Simulated Annealing	378
B.4 The Annealing Schedule	378
B.5 Killing Simulations	379
B.6 the Proposal	380
C Thermodynamic Integration	381
D McMC Values Report	385
E Writing Fortran/C Models	391
E.1 Model Subroutines, No Marginalization	391
E.2 The Parameter File	394
E.3 The Subroutine Interface	396
E.4 The Subroutine Declarations	398
E.5 The Subroutine Body	399
E.6 Model Subroutines With Marginalization	400
F the Bayes Directory Organization	405
G 4dfp Overview	407
H Outlier Detection	411
Bibliography	415

List of Figures

1.1	The Start Up Window	21
1.2	Example Package Interface	23
3.1	The Start Up Window	30
3.2	The Files Menu	31
3.3	The Load Image Selection Menu	33
3.4	The Packages Menu	37
3.5	The Working Directory Pull Down Menu	42
3.6	The Working Directory Po pup	43
3.7	The Settings Pull Down Menu	44
3.8	The McMC Parameters Po pup	44
3.9	The Edit Server Popup	45
3.10	The Submit Job Widget Group	48
3.11	The Server Widget Group	49
3.12	the Ascii Data viewer	50
3.13	the fid Data viewer	52
3.14	The Fid Data Viewer Display Type	53
3.15	The Fid Data Viewer the Options Menu	54
3.16	The Image Viewer	57
3.17	The Image Viewer Right Mouse Menu	58
3.18	The Prior Viewer	63
3.19	The Fid Model Viewer	66
3.20	The Data Model and Residuals	69
3.21	The Plot Information popup	70
3.22	The Posterior Probabilities	71
3.23	The Posterior Probabilities Vs Parameter Value	73
3.24	The Posterior Probabilities Vs Parameter Value a Skewed Example	74
3.25	The Expected Log Likelihood	76
3.26	The Scatter Plots	77
3.27	The Log Probability Plot	79
3.28	The Text Results Viewer	81
3.29	The Bayes Condensed File	84
3.30	Fortran/C Model Viewer	87
3.31	Fortran/C Model Viewer	88

4.1	Frequency Estimation Using The DFT	104
4.2	Aliases	105
4.3	Nonuniformly Nonsimultaneously Sampled Sinusoid	119
4.4	Alias Spacing	120
4.5	Which Is The Critical Time	122
4.6	Example, Frequency Estimation	123
4.7	Estimating The Sinusoids Parameters	125
5.1	the Exponential interface	130
6.1	the Unknown Exponential interface	136
6.2	The Distribution of Models	141
6.3	Exponential Probability for the Model	142
7.1	the Inversion Recovery interface	144
8.1	Bayes Analyze Interface	148
8.2	Bayes Analyze Fid Model Viewer	152
8.3	The Bayes Analyze File Header	170
8.4	The bayes.noise File	172
8.5	Bayes Analyze Global Parameters	175
8.6	Bayes Analyze Model File	176
8.7	Bayes Analyze Initial Model	178
8.8	Base 10 Logarithm Of The Odds	178
8.9	The bayes.output.nnnn Report	179
8.10	Bayes Analyze Uncorrelated Output	180
8.11	The bayes.probabilities.nnnn File	182
8.12	The bayes.log.nnnn File	185
8.13	The bayes.status.nnnn File	187
8.14	The bayes.model.nnnn File	188
8.15	The bayes.model.nnnn File Uncorrelated Resonances	189
8.16	Bayes Analyze Summary Header	189
8.17	The Summary2 Report	190
8.18	The Summary2 Report	191
9.1	The Big Peak/Little Peak Interface	198
9.2	The Time Dependent Parameters	208
10.1	The Bayes Metabolite Interface	210
10.2	Bayes Metabolite Viewer	212
10.3	Bayes Metabolite Probabilities List	217
10.4	The IPGD_D20 Metabolite	219
10.5	Bayes Metabolite IPGD_D20 Spectrum	220
10.6	Bayes Metabolite, The Fraction of Glucose	221
10.7	Glutamate Example Spectrum	223
10.8	Estimating The F_{c0} , y and F_{a0} Parameters	226
10.9	Bayes Metabolite, The Ethyl Ether Example	227

11.1 the Find Resonances interface	230
12.1 Diffusion Tensor Interface	238
12.2 Diffusion Tensor Parameter Estimates	246
12.3 Diffusion Tensor Posterior Probability For The Model	246
13.1 The Big Magnetization Package Interface	250
13.2 Big Magnetization Transfer Example Fid	252
13.3 Big Magnetization Transfer Expansion	253
13.4 Big Magnetization Transfer Peak Pick	254
14.1 Magnetization Transfer Interface	256
14.2 Magnetization Transfer Peak Pick	262
14.3 Magnetization Transfer Example Data	263
14.4 Magnetization Transfer Example Spectrum	264
15.1 Magnetization Transfer Kinetics Interface	268
15.2 Magnetization Transfer Kinetics Arrhenius Plot	274
15.3 Magnetization Transfer Kinetics Water Viscosity Table	275
16.1 Given Polynomial Order Package Interface	278
16.2 Given Polynomial Order Scatter Plot	284
17.1 Unknown Polynomial Order Interface	286
17.2 The Distribution of Models	290
17.3 Unknown Polynomial Order Package Posterior Probability	292
18.1 Errors In Variables Interface	296
18.2 Errors In Variables McMC Values File	302
19.1 the Behrens-Fisher interface	304
19.2 Behrens-Fisher Hypotheses Tested	305
19.3 Behrens-Fisher Console Log	315
19.4 Behrens-Fisher Status Listing	316
19.5 Behrens-Fisher McMC Values File, The Preamble	317
19.6 Behrens-Fisher McMC Values File, The Middle	318
19.7 Behrens-Fisher McMC Values File, The End	319
20.1 Enter Ascii Model Interface	322
21.1 Test Your Own Ascii Model Interface	330
22.1 Ascii Model Selection Interface	332
23.1 Absorption Model Images	334
23.2 Bayes Phase Interface	335
23.3 Bayes Phase Listing	341

24.1 Nonlinear Phasing Example	344
24.2 Nonlinear Phasing Interface	348
28.1 Image Pixels Example	362
A.1 Ascii Data File Format	368
D.1 The McMC Values Report Header	386
D.2 McMC Values Report, The Middle	387
D.3 The McMC Values Report, The End	388
E.1 Writing Models A Fortran Example	392
E.2 Writing Models A C Example	393
E.3 Writing Models, The Parameter File	395
E.4 Writing Models Fortran Declarations	399
E.5 Writing Models Fortran Example	402
E.6 Writing Models The Parameter File	403
G.1 The FD File Header	409
H.1 the Posterior Probability for the Number of Outliers	412
H.2 The Data, Model and Residual Plot With Outliers	414

List of Tables

8.1	Multiplet Relative Amplitudes	157
8.2	Bayes Analyze Models	173
8.3	Bayes Analyze Short Descriptions	186

Chapter 21

Test Your Own ASCII Model

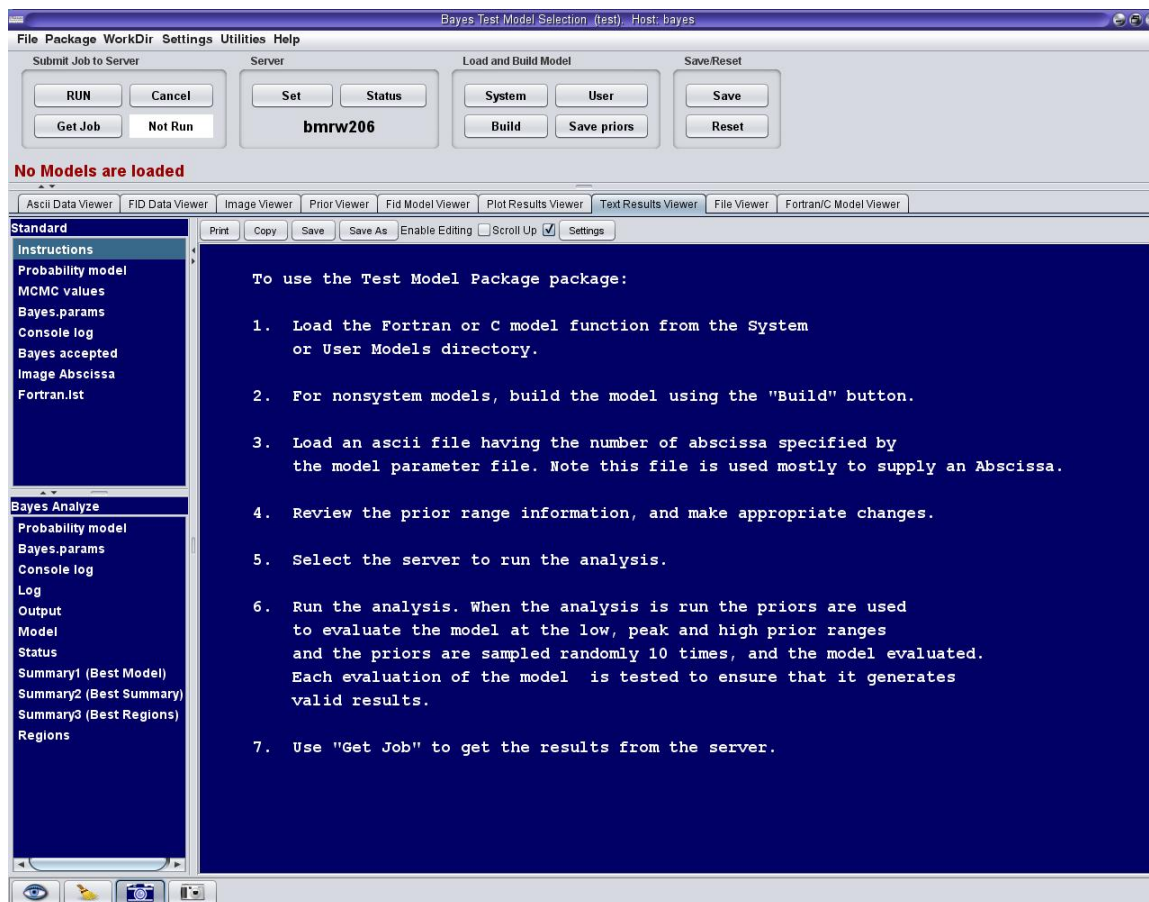


Figure 21.1: This is the interface to a package that lets you test your Ascii models. Sometimes, after building an Ascii model it will fail and tracking down the problem is very difficult because the models are not run on your local machine. This package attempts to help with testing Ascii Models. After loading a model and some data, this package will do everything it can to make the model malfunction. For more on the actual calculations and the widgets see the text.

Bibliography

- [1] Bayes, Rev. T. (1763), “An Essay Toward Solving a Problem in the Doctrine of Chances,” *Philos. Trans. R. Soc. London* **53**, pp. 370-418; reprinted in *Biometrika* **45**, pp. 293-315 (1958), and *Facsimiles of Two Papers by Bayes*, with commentary by W. Edwards Deming, New York, Hafner, 1963.
- [2] Bretthorst, G. Larry (1988), “Bayesian Spectrum Analysis and Parameter Estimation,” in *Lecture Notes in Statistics*, **48**, J. Berger, S. Fienberg, J. Gani, K. Krickenberg, and B. Singer (eds), Springer-Verlag, New York, New York.
- [3] Bretthorst, G. Larry (1990), “An Introduction to Parameter Estimation Using Bayesian Probability Theory,” in *Maximum Entropy and Bayesian Methods*, Dartmouth College 1989, P. Fougère ed., Kluwer Academic Publishers, Dordrecht the Netherlands, pp. 53-79.
- [4] Bretthorst, G. Larry (1990), “Bayesian Analysis I. Parameter Estimation Using Quadrature NMR Models” *J. Magn. Reson.*, **88**, pp. 533-551.
- [5] Bretthorst, G. Larry (1990), “Bayesian Analysis II. Signal Detection And Model Selection” *J. Magn. Reson.*, **88**, pp. 552-570.
- [6] Bretthorst, G. Larry (1990), “Bayesian Analysis III. Examples Relevant to NMR” *J. Magn. Reson.*, **88**, pp. 571-595.
- [7] Bretthorst, G. Larry (1991), “Bayesian Analysis. IV. Noise and Computing Time Considerations,” *J. Magn. Reson.*, **93**, pp. 369-394.
- [8] Bretthorst, G. Larry (1992), “Bayesian Analysis. V. Amplitude Estimation for Multiple Well-Separated Sinusoids,” *J. Magn. Reson.*, **98**, pp. 501-523.
- [9] Bretthorst, G. Larry (1992), “Estimating The Ratio Of Two Amplitudes In Nuclear Magnetic Resonance Data,” in *Maximum Entropy and Bayesian Methods*, C. R. Smith et al. (eds.), pp. 67-77, Kluwer Academic Publishers, the Netherlands.
- [10] Bretthorst, G. Larry, (1993), “On The Difference In Means,” in *Physics & Probability Essays in honor of Edwin T. Jaynes*, W. T. Grandy and P. W. Milonni (eds.), pp. 177-194, Cambridge University Press, England.
- [11] Bretthorst, G. Larry (1996), “An Introduction To Model Selection Using Bayesian Probability Theory,” in *Maximum Entropy and Bayesian Methods*, G. R. Heidbreder, ed., pp. 1-42, Kluwer Academic Publishers, Printed in the Netherlands.

- [12] Bretthorst, G. Larry (1999), “The Near-Irrelevance of Sampling Frequency Distributions,” in *Maximum Entropy and Bayesian Methods*, W. von der Linden *et al.* (eds.), pp. 21-46, Kluwer Academic Publishers, the Netherlands.
- [13] Bretthorst, G. Larry (2001), “Nonuniform Sampling: Bandwidth and Aliasing,” in *Maximum Entropy and Bayesian Methods in Science and Engineering*, Joshua Rychert, Gary Erickson and C. Ray Smith *eds.*, pp. 1-28, American Institute of Physics, USA.
- [14] Bretthorst, G. Larry, Christopher D. Kroenke, and Jeffrey J. Neil (2004), “Characterizing Water Diffusion In Fixed Baboon Brain,” in *Bayesian Inference And Maximum Entropy Methods In Science And Engineering*, Rainer Fischer, Roland Preuss and Udo von Toussaint *eds.*, AIP conference Proceedings **735**, pp. 3-15.
- [15] Bretthorst, G. Larry William C. Hutton, Joel R. Garbow, Joseph J.H. Ackerman, (2005), “Exponential parameter estimation (in NMR) using Bayesian probability theory,” *Concepts in Magnetic Resonance*, 27A, Issue 2, pp. 55-63.
- [16] Bretthorst, G. Larry William C. Hutton, Joel R. Garbow, Joseph J.H. Ackerman, (2005), “Exponential model selection (in NMR) using Bayesian probability theory,” *Concepts in Magnetic Resonance*, 27A, Issue 2, pp. 64-72.
- [17] Bretthorst, G. Larry, William C. Hutton, Joel R. Garbow, Joseph J.H. Ackerman, (2005), “How accurately can parameters from exponential models be estimated? A Bayesian view,” *Concepts in Magnetic Resonance*, 27A, Issue 2, pp. 73-83.
- [18] Bretthorst, G. Larry, W. C. Hutton, J. R. Garbow, J. J. H. Ackerman, (2008), “High Dynamic Range MRS Time-Domain Signal Analysis,” *Magn. Reson. in Med.*, **62**, pp. 1026-1035.
- [19] Chandramouli, Visvanathan, Karin Ekberg, William C. Schumann, Satish C. Kalhan, John Wahren, and Bernard R. Landau (1997), “Quantifying gluconeogenesis during fasting,” *American Journal of Physiology*, **273**, pp. H1209-H1215.
- [20] Cox R. T. (1961), “The Algebra of Probable Inference,” Johns Hopkins Univ. Press, Baltimore.
- [21] d’Avignon, André G. Larry Bretthorst, Marlyn Emerson Holtzer, and Alfred Holtzer (1998), “Site-Specific Thermodynamics and Kinetics of a Coiled-Coil Transition by Spin Inversion Transfer NMR,” *Biophysical Journal*, **74**, pp. 3190-3197.
- [22] d’Avignon, André G. Larry Bretthorst, Marlyn Emerson Holtzer, and Alfred Holtzer, (1999), “Thermodynamics and Kinetics of a Folded-Folded Transition at Valine-9 of a GCN4-Like Leucine Zipper,” *Biophysical Journal*, **76**, pp. 2752-2759.
- [23] Gilks, W. R., S. Richardson and D. J. Spiegelhalter (1996), “Markov Chain Monte Carlo in Practice,” Chapman & Hall, London.
- [24] Goggans, Paul M. and Ying Chi (2004), “Using Thermodynamic Integration to Calculate the Posterior Probability in Bayesian Model Selection Problems,” in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering: 23rd International Workshop*, Volume 707, pp. 59-66.

- [25] Holtzer, Marlyn Emerson, G. Larry Bretthorst, D. André d'Avignon, Ruth Hogue Angelette, Lisa Mints, and Alfred Holtzer (2001), "Temperature Dependence of the Folding and Unfolding Kinetics of the GCN4 Leucine Zipper via ^{13}C alpha-NMR," *Biophysical Journal*, **80**, pp. 939-951.
- [26] Jaynes, E. T. (1968), "Prior Probabilities," *IEEE Transactions on Systems Science and Cybernetics*, SSC-4, pp. 227-241; reprinted in [29].
- [27] Jaynes, E. T. (1978), "Where Do We Stand On Maximum Entropy?" in *The Maximum Entropy Formalism*, R. D. Levine and M. Tribus Eds., pp. 15-118, Cambridge: MIT Press, Reprinted in [29].
- [28] Jaynes, E. T. (1980), "Marginalization and Prior Probabilities," in *Bayesian Analysis in Econometrics and Statistics*, A. Zellner, ed., North-Holland Publishing Company, Amsterdam; reprinted in [29].
- [29] Jaynes, E. T. (1983), "Papers on Probability, Statistics and Statistical Physics," a reprint collection, D. Reidel, Dordrecht the Netherlands; second edition Kluwer Academic Publishers, Dordrecht the Netherlands, 1989.
- [30] Jaynes, E. T. (1957), "How Does the Brain do Plausible Reasoning?" unpublished Stanford University Microwave Laboratory Report No. 421; reprinted in *Maximum-Entropy and Bayesian Methods in Science and Engineering* **1**, pp. 1-24, G. J. Erickson and C. R. Smith Eds., 1988.
- [31] Jaynes, E. T. (2003), "Probability Theory—The Logic of Science," edited by G. Larry Bretthorst, Cambridge University Press, Cambridge UK.
- [32] Jeffreys, Harold Sir (1939), "Theory of Probability," Oxford Univ. Press, London; Later editions, 1948, 1961.
- [33] Jones, John G. (2001), Michael A. Solomon, Suzanne M. Cole, A. Dean Sherry, Craig R. Malloy "An integrated ^2H and ^{13}C NMR study of gluconeogenesis and TCA cycle flux in humans," *American Journal of Physiology, Endocrinology, and Metabolism*, **281**, pp. H848-H856.
- [34] Kotyk, John, N. G. Hoffman, W. C. Hutton, G. Larry Bretthorst, and J. J. H. Ackerman (1992), "Comparison of Fourier and Bayesian Analysis of NMR Signals. I. Well-Separated Resonances (The Single-Frequency Case)," *J. Magn. Reson.*, **98**, pp. 483-500.
- [35] Laplace, Pierre Simon (1814), "A Philosophical Essay on Probabilities," John Wiley & Sons, London, Chapman & Hall, Limited 1902. Translated from the 6th edition by F. W. Truscott and F. L. Emory.
- [36] Lartillot, N. and H. Philippe (2006), "Computing Bayes Factors Using Thermodynamic Integration," *Systematic Biology*, **55**(2) pp. 195-207.
- [37] Le Bihan, D. (1985), E. Breton, "Imagerie de diffusion in-vivo par rsonance," *C R Acad Sci (Paris)* **301** (15) pp. 1109-1112.
- [38] Lomb, N. R. (1976), "Least-Squares Frequency Analysis of Unevenly Spaced Data," *Astrophysical and Space Science*, **39**, pp. 447-462.

- [39] Loredó, T. J. (1990), “From Laplace To SN 1987A: Bayesian Inference In Astrophysics,” in *Maximum Entropy and Bayesian Methods*, P. F. Fougere (ed), Kluwer Academic Publishers, Dordrecht, The Netherlands.
- [40] Malloy, Craig R., A. Dean Sherry, F. Mark H. Jeffrey (1988), “Evaluation of Carbon Flux and Substrate Selection through Alternate Pathways Involving the Citric Acid Cycle of the Heart by ^{13}C NMR Spectroscopy,” *Journal of Biological Chemistry*, Vol. 263, No. 15, pp. 6964-6971.
- [41] Malloy, Craig R., A. Dean Sherry, F. Mark H. Jeffrey (1990), “Analysis of tricarboxylic acid cycle of the heart using ^{13}C isotope isomers,” *American Journal of Physiology*, **259**, pp. H987-H995.
- [42] Merboldt, K., Hanicke, W., Frahm, J. (1969), “Self-diffusion NMR imaging using stimulated echoes,” *Journal of Magnetic Resonance* **64** (3) pp. 479-486.
- [43] Metropolis, Nicholas, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller (1953), “Equation of State Calculations by Fast Computing Machines,” *Journal of Chemical Physics*. The previous link is to the American Institute of Physics and if you do not have access to Science Sitations you may not be able to retrieve this paper.
- [44] Neal, Radford M. (1993), “Probabilistic Inference Using Markov Chain Monte Carlo Methods,” technical report CRG-TR-93-1, Dept. of Computer Science, University of Toronto.
- [45] Neil, Jeffrey J., and G. Larry Bretthorst (1993), “On the Use of Bayesian Probability Theory for Analysis of Exponential Decay Data: An Example Taken from Intravoxel Incoherent Motion Experiments,” *Magn. Reson. in Med.*, **29**, pp. 642-647.
- [46] Nyquist, H. (1924), “Certain Factors Affecting Telegraph Speed,” *Bell System Technical Journal*, **3**, pp. 324-346.
- [47] Nyquist, H., (1928), “Certain Topics in Telegraph Transmission Theory,” *Transactions AIEE*, **3**, p. 617-644.
- [48] Press W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannary (1992), “Numerical Recipes The Art of Scientific Computing Second Edition,” Cambridge University Press, Cambridge UK.
- [49] Scargle, J. D. (1982), “Studies in Astronomical Time Series Analysis II. Statistical Aspects of Spectral Analysis of Unevenly Sampled Data,” *Astrophysical Journal*, **263**, pp. 835-853.
- [50] Scargle, J. D. (1989), “Studies in Astronomical Time Series Analysis. III. Fourier Transforms, Autocorrelation and Cross-correlation Functions of Unevenly Spaced Data,” *Astrophysical Journal*, **343**, pp. 874-887.
- [51] Schuster, A., (1905), “The Periodogram and its Optical Analogy,” *Proceedings of the Royal Society of London*, **77**, p. 136-140.
- [52] Shannon, C. E. (1948), “A Mathematical Theory of Communication,” *Bell Syst. Tech. J.* **27**, pp. 379-423.
- [53] Shore J. E., R. W. Johnson (1981), “Properties of cross-entropy minimization,” *IEEE Trans. on Information Theory*, **IT-27**, No. 4, pp. 472-482.

- [54] Shore J. E., R. W. Johnson (1980), "Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy," *IEEE Trans. on Information Theory*, **IT-26**, No. 1, pp. 26-37.
- [55] Sivia, D. S. and J. Skilling (2006), "Data Analysis: A Bayesian Tutorial," Oxford University Press, USA.
- [56] Stejskal, E. O., Tanner, J. E. (1965), "Spin Diffusion Measurements: Spin Echoes in the Presence of a Time-Dependent Field Gradient." *Journal of Chemical Physics* **42** (1), pp. 288-292.
- [57] Taylor, D. G., Bushell, M. C. (1985), "The spatial mapping of translational diffusion coefficients by the NMR imaging technique," *Physics in Medicine and Biology* **30** (4), pp. 345-349.
- [58] Tribus, M. (1969), "Rational Descriptions, Decisions and Designs," Pergamon Press, Oxford.
- [59] Woodward, P. M. (1953), "Probability and Information Theory, with Applications to Radar," McGraw-Hill, N. Y. Second edition (1987); R. E. Krieger Pub. Co., Malabar, Florida. 1990.
- [60] Zellner, A. (1971), "An Introduction to Bayesian Inference in Econometrics," John Wiley and Sons, New York.