

Bayesian Analysis Users Guide
Release 4.00, Manual Version 1

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Chapter 2

Installing the Software

This Chapter will walk you through the process of setting up a Bayesian software server. The server contains all of the software needed to run the various analysis and once installed, your server is an independent standalone installation that does not communicate with Washington university, except when you download or check for updates. Here are the steps needed to setup a server:

Pick the machine that is to act as the server. Lets call this machine “your.server.net.” The server(s) can be a multi-core LinuxPC, either 32 or 64 bit, running CentOS 4.1 or higher, or a Sun system running Solaris 9 or 10.

Both Java (6.0 or higher) and javaws must be accessible to “your.server.net.” Note for the LinuxPC uses, the Java shipped with CentOS is OpenJDK and this Java version does not come with javaws. To fix this problem, download and install Sun Java.

The Fortran and C compilers are not strictly required to run this software, but the software is much more usable when these are installed. For this reason, I would strongly suggest users install both Fortran and C compilers. The software supports Intel, Gnu and Sun compilers. The Intel compilers cost a few hundred dollars, the Gnu and Sun compilers are free. Note that while the software supports Sun compilers on Solaris, it does not support these compilers on Linux systems.

Create an account on “your.server.net” that is to be configured to run the software. This account should be dedicated to running the Bayesian Analysis software and should not be used by anyone for any other purpose. For arguments sake, lets assume this account is named “bayes”, although it can have any name.

Login to the “bayes” account on “your.server.net” and configure the path variable so that both Java and javaws are in your path.

Run the following command:

```
javaws http://bayes.wustl.edu/ServerSoftware/launch.jnlp
```

When this command executes, you will get the popup shown in Fig. 2.1. This Java application will bring up a configuration window with the software's best guess as to what is needed in the various configuration parameters. Review these settings and correct any that are not set correctly.

Clicking the install button will cause the install kit to download the software from bayes.wustl.edu and then install that software in the top level directory of the bayes account. The software runs as user bayes and does not have write permission outside of the bayes account. The compressed tar file that is downloaded is between 35 and 45 megabytes depending your server hardware, so downloading this file could take a few minutes.

After the server installation kit completes, it displays a list of commands that must be executed as root. These commands will copy the Apache server configuration files to the appropriate system directory, make the server's home directory and install the start/stop files for the server. These commands can be cut and pasted, so you do not have to retype them.

Your sever should be up and running and you should have a fully functional installation of the Bayesian Analysis software. To check, entering "http://your.server.net:8080" in a web browser should bring up the default apache server page; where your.sever.net is the name of your server, and you should replace 8080 by the port number used in the installation. If the apache server home page appears, everything is working, if not something has gone wrong in the installation and you should contact us for assistance.

You should be able to run the client interface on any machine that has Java and JavaWS installed. We routinely run the interface on Sun Solaris, MacOS, CentOS and WindowsPCs. To run the interface, issue the command:

```
javaws http://your.server.net:8080/Bayes/launch.jnlp
```

from a command prompt, where you must supply your server name and port number. Additionally, this command can be placed in a shortcut to facilitate starting the interface. Running this command should result in the interface being displayed on your client machine. If you wish to create multiple servers, repeat all of the steps including copying the setup commands as root for each of your servers.

Finally, the Bayesian Analysis software does not have an update feature. That is to say, when we distribute new software, one simply reruns the installation procedure to install the updates. So, for example, if you installed the software but did not configure either Fortran or C compilers, then to change this you would simply rerun the installation procedure and select the appropriate compilers. When the installation is done, you system should then support compilers. If you have any questions or comment please contact me: [Email: larry@bayes.wustl.edu](mailto:larry@bayes.wustl.edu)

Figure 2.1: Installation Kit For The Bayesian Analysis Software

Bayesian Server Software Installation/Update

File Help

Installation Info

**Bayesian Analysis Of Common NMR Problems
version 4.10 is being installed**

Host Configuration

Hostname	192.168.122.1
Host IP	192.168.122.1
Host OS	CentOS release 6.4 (Final)
Hardware	x86_64
User	larry
Group	larry
Shell	/bin/csh
Home Dir	/home/larry

Apache Server

CentOS/Sun Ubuntu/Debian

Server Root	/etc/httpd	Set
Doc. Root	/var/www/html	Set
HTTPD	/usr/sbin/httpd	Set
PID File	/etc/httpd/run/httpd.pid	Set
Log Dir	/etc/httpd/logs	Set
Config File	/etc/httpd/conf/httpd.conf	Set
Start/Stop File	/etc/rc5.d/K15httpd	Set

Server Parameters

Port	8080
Queue	None
Email	

Use Server Passwords
 Email Update Notifications
 Setup Apache Server

QUIT INSTALL

Figure 2.1: When the `javaws` command executes, it looks over your computer and makes its best guesses as to how to configure the software. After determining these guesses, this window is displayed so that you can correct and fill in some additional information. After adding compilers, hit the “Install” button to load the software. The install program will download and install the software. When it completes it pops up a window that contains a number of commands that must be run as root. These commands copy the configuration files to their proper location.

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