

BOOK REVIEW *

Methods of Statistical Physics

A. I. Akhiezer and S. V. Peletminskii (Trans. M. Schukin)

448 pp., Pergamon Press, New York, 1981, \$54.00

Statistical Mechanics is a field of sharp dichotomies, with few attempts to strike a balance. Some writers show an exclusive preoccupation with minutiae of mathematical rigor to the neglect of physical considerations; and vice versa. The same is true with regard to emphasis on principles vs. applications, Boltzmannian distribution functions vs. Gibbsian canonical ensembles, stochastic models vs. correlation functions, philosophical interpretation vs. pragmatic prediction, etc.

As noted in a Foreward by N. N. Bogoliubov, this work is unique in that the authors try to strike a reasonable balance in these respects. In this reviewer's opinion, they come closer than anyone else to achieving this, but miss a point of basic understanding needed to bridge the most fundamental dichotomy.

The many applications have a neat and elegant quality. As one would expect from other well-known works of Akhiezer, the treatment of macroscopic electrodynamics is particularly thorough. The derivation of macroscopic hydrodynamics extracts a surprising amount of information from Galilean invariance; and some of the special properties of superfluids are then seen as resulting from failure of Galilean invariance. The Wigner distribution function, hitherto a rather mysterious and unwieldy thing defined on the $6N$ -dimensional phase space, becomes by use of quantized wave functions a Wigner distribution operator in ordinary position-velocity space, a much simpler and more useful quantity.

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The applications are so interesting, useful, and well presented that it is a temptation to concentrate entirely on them, passing over the less happy discussion of fundamentals. However, to do so would run counter to the purpose of the work as indicated by the authors in the Preface. The applications, valuable as they may be, are given to explicate certain general principles that the authors adopt as fundamental to statistical mechanics. It is really these principles that are being expounded, and we owe it to the authors to examine the work in that light.

Indeed, the useful applications of statistical mechanics are connected rather loosely to those fundamentals. In any particular application, the important and undoubtedly correct results usually turn out to be derivable from many different philosophical viewpoints. The danger of a too narrow viewpoint is not so much that one will make a wrong prediction, but rather that he will think that the validity of his result depends on extraneous assumptions that are actually unnecessary. Therefore, instead of asking which viewpoint is "correct"--a matter of personal opinion--it is better to ask which viewpoint leads us to the most general results with the fewest assumptions--a matter of demonstrable fact.

The three general principles that the authors expound are the "contraction of distribution functions", the "attenuation of correlations", and the "ergodic relation". According to the first, after a short initial "kinetic phase" a joint probability distribution for the positions and momenta of n particles becomes a functional of (i.e., is determined by) the Boltzmann single-particle distribution $f(x,p,t)$. This was advanced by Bogoliubov in 1947 as a tentative conjecture; the intervening years seem to have brought, if not proof, at least confidence; for it is now asserted as a general principle.

However, this is not a criticism; for generally by a "principle" of statistical mechanics is meant some proposition that one wants to adopt, but cannot prove (for if it could be proved, then it would be a result, not a principle). Of course, this necessarily entails the risk that one's principles will be disproved.

The second principle is that as the separation of particles increases, their probability distributions become independent (in particular, uncorrelated). There seems also to be a presumption, at first glance quite plausible, that the range of correlations is short, of the order of the range of forces (p. 12), or at most a few times the mean free path. However, some caution is needed in assuming this. For example, from the theory of response functions given in Chapter 4, the acoustic Green's function is $(kT)^{-1}$ times the space-time correlation function of the air pressure fluctuations, $\langle \delta P(x,t) \delta P(x',t') \rangle$. It follows that, if a student in the back of a lecture hall can hear the teacher's voice, it is only because thermal pressure fluctuations at the student's ear and the teacher's mouth are correlated, over a distance of perhaps 10^9 mean free paths. If the distance required for attenuation of correlations is larger than the size of the macroscopic system under study, then the principle does not seem to have much content.

The third principle seeks to deal with a dilemma of interpretation that haunts us throughout the work, starting literally on page 1. Over and over again we find the statement that a system "makes a transition into a state of statistical equilibrium". The quantum-mechanical version of this is, notationally, easier to describe. Given an initial "statistical operator" or density matrix $\rho(0)$, common teaching holds that its time

development is given by the Schrödinger equation of motion:

$$\rho(t) = \exp(-iHt)\rho(0)\exp(iHt) \quad (1)$$

On the other hand, equally common teaching holds that a system in thermal equilibrium at temperature T is described by a Gibbsian canonical distribution:

$$\rho_C \propto \exp(-H/kT) \quad (2)$$

Suppose, then, that a system with Hamiltonian H , in an initial nonequilibrium state described by $\rho(0)$, is left to itself and comes eventually to thermal equilibrium. If one believes both of those common teachings, he seems forced to the conclusion that dynamical evolution (1) must in the course of time take us to the "state of statistical equilibrium" (2):

$$\rho(t) \rightarrow \rho_C \quad (3)$$

But it is trivial to prove that (3) cannot, in general, be true. For (1) is a unitary transformation, and so not only is the information entropy $S_I = -k \text{Tr}(\rho \ln \rho)$ a constant, each individual eigenvalue of $\rho(t)$ is a constant of the motion. If the eigenvalues of the initial $\rho(0)$ are not the same as those of ρ_C , then no unitary transformation can carry $\rho(0)$ into ρ_C .

The difficulty was not seen so clearly in equilibrium theory, where one simply postulated the canonical form (2) and never paid much attention to (1). But as soon as we have to explain how a system manages to get into the equilibrium state (2), we have the basic dilemma of irreversible statistical mechanics. If we deny the validity of (1) we are denying that the system obeys the Schrödinger equation. If we deny the validity of (2) we are denying experimental facts. Yet it is a mathematical theorem

that in general (1) and (2) are incompatible. Each writer on the subject must find some way around this difficulty, most try simply to obscure it. Akhiezer and Peletminskii are refreshingly clear and forthright on this point; they simply ignore the mathematical theorem and assert the validity of (3) [their equation (2.4.24)] as an "ergodic relation". Equilibrium is achieved by fiat.

In this reviewer's opinion, a basic point of understanding is missed here, although it was recognized clearly by Khinchin about forty years ago. In trying to bridge the dichotomy between (1) and (2), to demand that the distributions themselves become the same is far stronger than necessary, and is almost always untrue. It is sufficient to show that their physical predictions, for the particular macroscopic quantities of interest, become the same. It may well be that the principle of contracted distributions would have helped in demonstrating this, but for intervention of the extraneous "ergodic relation".

In summary, the work has a beautiful and impressive collection of applications, which teachers of advanced statistical mechanics will want to use. The discussion of fundamentals is dated, and would need much revision before it would be suitable as a modern textbook.

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Footnote: E. T. Jaynes has for 30 years been engaged in writing and research on foundations of statistical mechanics and on general statistical theory.