Book Reviews

Principles of Statistical Mechanics—The Information Theory Approach, Amnon Katz (W. H. Freeman and Co., San Francisco, Calif., 1967, 188 + iii pp., $6.50).

The introduction of any new method into science sets up a transient which requires several years to damp out. There is an initial overshoot of extravagant hopes and perhaps extravagant claims, followed by an undershoot of denials from those who have a vested interest in the previous state of affairs; and the ripples of overestimation and underestimation continue as we gradually build up a collective experience in applying it. Eventually, we reach the point where a sober reassessment is possible, and we can make objective judgments about just what the method can accomplish for us, and what it cannot. This point is seldom reached in less than ten years.

We have all watched with interest the large waves generated by Shannon's original introduction of information theory in 1948; and I think we would agree that, in their impact on engineering, the ripples required about twelve to fifteen years to damp out. A minor transient was added to this in 1957, with the suggestion by this reviewer that the principles of information theory could be applied to problems of physics, leading to a generalization and simplification of the statistical mechanics of Gibbs. The basic idea is extremely simple: the probability distribution over microstates which has maximum entropy subject to constraints representing our macroscopic information about a system, provides the most "honest" description of our state of knowledge about it; and so predictions based on that distribution provide a useful form of inductive reasoning.

The rather tiny ripples generated by this have been running their course largely in the areas of physics and mechanical engineering, and more recently in biology; and have perhaps not been evident at all in electrical engineering (although I believe they will soon put in an appearance there). In any event, they have by no means died out, and we are only now approaching the point where sober reassessment can be attempted; perhaps we will have it by 1970.

Until this sober reassessment is an accomplished fact all expositions of the information theory approach to statistical mechanics are of necessity provisional, incomplete, and in part incorrect; they are part of the ripples and not part of the mature subject. To say this is in no way to deprecate them; for they represent necessary stages on the way to maturity, just as a toddler must necessarily stumble and fall a few times before learning how to walk. Every worthwhile development in science must pass through such a phase; and the book under review represents an important part of it.

In the preface, the author attempts to make an assessment of the maximum-entropy principle in statistical mechanics, and concludes: "Information Theory makes statistical mechanics a complete theory by setting up rules for asking any question. It contributes nothing, however, to the technical problem of getting the answer." At the end of Chapter I, he continues, "What it does contribute is the unification of all of statistical mechanics into one consistent and well-defined theory."

I consider this a fair, if slightly inaccurate, statement of the scope of the method as it existed in 1958; and the book itself to be a sound and useful exposition of the theory at that stage of its development. It is, therefore, strongly recommended for any reader who wishes to understand the beginnings of this theory, without having to read all of my own philosophical ramblings and false starts.

In 1958, the information theory approach could be justly claimed to be only a useful pedagogical trick, by which known results could be derived in a shorter and simpler way than previously. While this alone is perhaps sufficient to justify its existence, it is clear that the real test of the method must be its ability to produce new results, which had not been found by previous methods. More specifically, the theory at that stage accomplished a unification of equilibrium statistical mechanics, but the proper generalization to nonequilibrium problems had not yet been found; and so the theory expounded in this book is hardly a unification of all of statistical mechanics.

In fact, it required several more years of mathematical exploration and physical meditation before this generalization was finally accomplished in 1963; and an essential principle for construction of nonequilibrium ensembles was found only in 1967, as described in the Washington University doctoral thesis of William C. Mitchell. For a number of reasons, the 1963 development has not yet been fully published; however, the underlying ideas and basic equations, shorn of all unnecessary mathematical details, appear in the chapter entitled, "Foundations of Probability Theory and Statistical Mechanics," (in *Delaware Seminar in the Foundations of Physics*, M. Bunge, Ed. Berlin: Springer, 1967). We will note below to what extent this theory now does contribute to the "technical problem of getting the answer."

The first two chapters of Katz's book provide a short introduction to the notions of statistical mechanics, and of entropy as an information measure. Unfortunately, the discussion of entropy of a continuous distribution repeats a technical error which has now persisted in the literature for twenty years. It is important that this error be recognized and corrected, so let us dwell on it a bit.

Shannon gave the basic uniqueness theorem establishing the quantity $H_d = -\sum p_i \log p_i$ as the information measure for a discrete probability distribution. But in the continuous case he simply assumed, without any derivation, that the analogous expression was

$$H' = -\int p(x) \log p(x)\, dx$$

and got into some trouble because H' lacks invariance under a change of variables $x \to y(x)$. In Shannon's applications the trouble was minor, and did not affect the final conclusions. In the application to statistical mechanics, however, use of H' leads to predictions that depend critically on the choice of variables we make.

Now, as this reviewer pointed out in his 1962 Brandeis lectures (in *Statistical Physics*, K. W. Ford, Ed. New York: W. A. Benjamin, Inc., 1963, ch. 4), in the absence of any more direct proof, the only criterion we have for finding the correct information measure in the continuous case is to pass to the limit from a discrete distribution, where Shannon's uniqueness theorem applies. If this limiting operation is carried out in a sufficiently careful and general manner, we find that the proper continuous information measure is not H', but

$$H_c = -\int p(x) \log [p(x)/m(x)]\, dx$$

where $m(x)$ is an "invariant measure" function, determined by the limiting density of discrete points. H_c is invariant under coordinate transformations, and it is easily shown that, in maximum-entropy inference based on H_c, not only our final conclusions, but also our Lagrange multipliers and partition functions, are all invariant so that they acquire definite physical meanings.

In many sample spaces or parameter spaces which are not the result of any obvious limiting process, the proper invariant measure can be found by methods of group theory; the group of coordinate transformations which convert the problem into a physically equivalent one determines a functional equation whose solution is the required $m(x)$. Application of statistical mechanics to systems where the equations of motion are not of Hamiltonian form cannot be done at all until these points are recognized. The author's adherence to H' leads to no definite theory of inductive inference; the resulting formalism cannot be applied safely in any problem except the one he considers, where the answers were already known.

Chapters 3–7 give the application of this principle to equilibrium statistical mechanics. This is all standard material, since in this case one is led to just the same equations already given by Gibbs (even here, however, as was noted in the Brandeis lectures, but not in this book, the information theory viewpoint leads to a greater flexibility in our choice of the basic variables).

The only place where new results are possible is in the extension to nonequilibrium problems, where previously no definite, unambiguous theory existed. However, the author's discussion of small deviations from equilibrium (transport coefficients, etc.) in Chapter 8, is conducted in a way that owes nothing at all to the information theory approach. It reproduces only the results which had been found by Onsager, Casimir, Green, Kirkwood, Callen, Kubo, Mori, and others prior to 1958, and which made no use of information theory. A more complete account of this development can be found in R. Kubo's lectures (*1958 Boulder Lectures in Theoretical Physics*, Vol. 1, W. Brittin and L. Dunham, Eds. New York: Interscience 1959). At this point, let us turn back to that "technical problem of getting the answers."

In all of this early work, one was troubled with the so-called "induction time" and "plateau" phenomena. It was necessary to carry out some kind of coarse-graining, usually a time or space-time average, before equations of the desired phenomenological form (diffusion current proportional to density gradient, etc.) emerged. In this work, one first set up an ensemble usually called the "local equilibrium" or "frozen-state" ensemble, which described spatial variations of temperature, particle density, etc. But when we try to calculate the resulting fluxes (heat flow, etc.) or rates of relaxation to equilibrium from this ensemble, we find to our dismay that the result is identically zero.

Mathematically, it was found necessary to integrate the equations of motion forward for a short "induction time" before the irreversible process gets going; and uncertainties about just how long we must integrate before reaching the conjectured "plateau" values led to ambiguities in the final formulas for transport coefficients. Perhaps even worse from a practical standpoint, the introduction of coarse-graining has the consequence that one cannot treat general problems; but is limited to the quasi-stationary or long-wavelength limit. This has been a famous difficulty in the theory of irreversible processes, well recognized for over twenty years.

It is precisely at this point that the information theory approach can make one of its most important contributions to the "technical problem of getting the answers." The need for carrying out this forward integration and coarse-graining is, of course, merely a symptom of the fact that the initially chosen ensemble did not correctly represent the physical situation (or, more accurately stated, the correct range of microstates) in which we have the irreversible process. These operations are corrective measures which in some way compensate for the error in the initial ensemble.

But, if the theory were correctly set up in the first place, such artificial devices would not be necessary; the fluxes and transport coefficients could be calculated by direct quadratures over the initial ensemble. Furthermore, instead of mutilating the formalism by coarse-graining in order to obtain equations of a preconceived phenomenological form, we could tell from the theory the exact conditions under which this form is correct. However, before the introduction of the information theory viewpoint, nobody was able to understand the exact nature of this error, much less how to correct it; and it is not recognized in this book.

Here is the secret. Recall that in the information theory approach, we are given certain information and use the principle of maximum entropy to construct an ensemble corresponding to that information. But, we can equally well reason in the opposite direction; the entirely new insight into this problem is the realization that, given any proposed ensemble, it makes perfectly definite sense to ask, "*What is the information contained in this ensemble?*" It is the same as asking, "With respect to *which* constraints does this ensemble have maximum entropy?" To me, it is one of the most beautiful aspects of this theory that the mathematics works out so that the answer can be read off immediately, by mere inspection of any ensemble.

As soon as the problem was looked at in this way, the answer was obvious at a glance; a very essential piece of information was missing in the "frozen-state" ensemble. To restore it, one needs to incorporate information not only over a space region at one instant of time, but over a *space-time* region; and the partition function gets generalized to a partition functional over functions defined in this region. This insight into how to correct the "frozen-state" ensemble had the effect of opening the flood gates; immediately, both old and new formulas for transport coefficients poured forth as fast as one could write. Because the forward integration was done with, one could specify the conditions under which Kubo's formulas were exact; and give corrections for other conditions. Because the coarse-graining was done with, we were no longer restricted to quasi-stationary processes like diffusion; with equal ease, the formalism yielded general formulas for rapid processes such as ultrasonic attenuation. With further mathematical development, I believe it will be possible to find equally general treatments of highly nonlinear phenomena, such as shock waves.

It is in the areas just indicated that the real power of the information theory approach is found. Until the full theory has been written up, and a few more applications worked out, an objective assessment of its usefulness and limitations will not be possible.

In summary, this book is a useful introduction; but the reader should be forewarned that the real justification of this approach appears only in further developments that begin where this book leaves off. Except for a few apparently original proofs, it contains no actual results that had not been published elsewhere; in most cases prior to 1958 and in much more complete form. The serious student of the subject will, therefore, be astonished, dismayed, and handicapped by the failure to provide even a single reference. There is not even any acknowledgement of the work—or indeed, the existence—of Gibbs and Shannon, who created the two streams of thought here fruitfully merged.

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